Name:____ **4/19/2017**

Math 392 Linear Algebra and Vector Analysis for Engineers **Midterm Exam**

1. (8 points) Evaluate the following line integral with respect to arc length, where C is the line segment from $(0, 0, 0)$ to $(1, 2, 3)$.

$$\int_C xe^{yz}\, ds$$

2. Let $\mathbf{F}(x, y) = (ax^2y + y^3 + 1)\vec{\imath} + (2x^3 + bxy^2 + 2)\vec{\jmath}$ be a vector field, where a and b are constants.

(a) (4 points) Find the values of a and b for which $\mathbf{F}$ is conservative.

(b) (4 points) For these values of a and b, find $f(x, y)$ such that $\mathbf{F} = \nabla f$.

(c) (4 points) Still using the values of a and b from part (a), compute $\int_C \mathbf{F} \cdot d\mathbf{r}$ along the curve C such that $x = e^t \cos t$, $y = e^t \sin t$, $0 \leq t \leq \pi$.

3. Verify that Green's theorem is true for the line integral

$$\int_C xy\, dx + x^2\, dy$$

where C is the triangle with vertices $(0,0), (1,0), (1,2)$ oriented counterclockwise. Do this by calculating

(a) (8 points) the line integral directly, and

(b) (8 points) the related double integral.

4. (8 points) Find the surface area of the part of the surface $z = x^2 + y^2$ with $1 \leq z \leq 2$.

5. Let C be the intersection curve of the surfaces

$$z = 3x \qquad \text{and} \qquad x^2 + y^2 = 1$$

oriented counterclockwise as seen from above. Calculate

$$\int_C (1 - 4z)\,dx + 2x\,dy + (1 - 5z)\,dz$$

(a) (8 points) directly as a line integral, and

(b) (8 points) as a double integral, by using Stoke's theorem.