Name: *ANSWER KEY* 2/15/2017

Math 392 Linear Algebra and Vector Analysis for Engineers Quiz 1

1. Give either a vector function or parametric equations for the tangent line to the curve

$$\vec{r}(t) = \langle 1 + 2\sqrt{t},\ t^3 - t,\ t^3 + t \rangle$$

at the point $(3, 0, 2)$.

LET $\vec{u}(t)$ BE A VECTOR FUNCTION FOR TANGENT LINE TO $\vec{r}(t)$ AT $\langle 3,0,2\rangle = \vec{r}(1)$.

$\vec{u}(t) = \vec{r}(1) + t\,\vec{r}\,'(1)$

$\vec{r}\,'(t) = \langle \frac{1}{\sqrt{t}}, 3t^2 - 1, 3t^2 + 1 \rangle$

$\vec{r}\,'(1) = \langle 1, 2, 4 \rangle$, SO

$$\boxed{\vec{u}(t) = \langle 3,0,2\rangle + t\langle 1,2,4\rangle = \langle 3+t, 2t, 2+4t\rangle}$$

$$\boxed{\text{i.e. } \begin{aligned} x(t) &= 3+t \\ y(t) &= 2t \\ z(t) &= 2+4t \end{aligned}}$$

(HW §10.8 #7)

2. Let C be the curve of intersection of the parabolic cylinder $x^2 = 2y$ and the surface $3z = xy$. Find the exact length of C from the origin to the point $(6, 18, 36)$.

PARAMETRIZE C: LET $x = t$. THEN $y = \frac{1}{2}t^2$ & $z = \frac{1}{6}t^3$

SO C IS DESCRIBED BY $\vec{r}(t) = \langle t, \frac{1}{2}t^2, \frac{1}{6}t^3 \rangle$

WITH $0 \le t \le 6$ $\left(\vec{r}(0) = \text{ORIGIN}, \vec{r}(6) = \langle 6, 18, 36\rangle\right)$.

WE HAVE $\vec{r}\,'(t) = \langle 1, t, \frac{1}{2}t^2 \rangle$ AND SO

$$|\vec{r}\,'| = \sqrt{(1)^2 + (t)^2 + (\tfrac{1}{2}t^2)^2} = \sqrt{1 + t^2 + \tfrac{1}{4}t^4}$$

$$= \sqrt{(1 + \tfrac{1}{2}t^2)^2} = 1 + \tfrac{1}{2}t^2$$

$$L = \int_0^6 |\vec{r}\,'(t)|\,dt = \int_0^6 \left(1 + \tfrac{1}{2}t^2\right)dt = \left(t + \tfrac{1}{6}t^3\right)\Big|_0^6$$

$$= 6 + 36 = \boxed{42}$$

(HW §10.8 #9)

3. Reparametrize the curve

$$\vec{r}(t) = \langle 2t,\ 1-3t,\ 5+4t\rangle$$

with respect to arc length measured from the point $(0,1,5)$ in the direction of increasing t.

$$\vec{r}\,'(t) = \langle 2, -3, 4\rangle \implies |\vec{r}\,'(t)| = \sqrt{(2)^2+(-3)^2+(4)^2} = \sqrt{29}$$

$$\therefore \quad s = \int_0^t |\vec{r}\,'(u)|\,du = \sqrt{29}\int_0^t dt = \sqrt{29}\,t$$

$$\implies t = \frac{1}{\sqrt{29}}\,s$$

$$\text{So} \quad \vec{r}(t) = \vec{r}\left(\frac{1}{\sqrt{29}}s\right) = \boxed{\left\langle \frac{2}{\sqrt{29}}s,\ 1-\frac{3}{\sqrt{29}}s,\ 5+\frac{4}{\sqrt{29}}s\right\rangle}$$

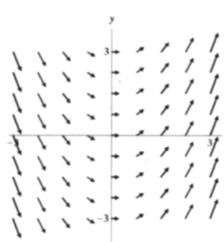
4. Match the following vector fields on $\mathbb{R}^2$ with their plots labeled I-IV.

(a) $\vec{F} = \langle x,\ 1\rangle$ III (ALL POINTING UP)

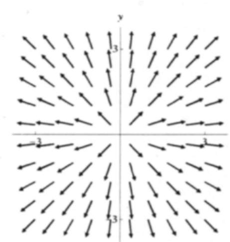
(b) $\vec{F} = \langle 1,\ x\rangle$ I (ALL POINTING RIGHT)

(c) $\vec{F} = \nabla f$, where $f(x,y) = x^2+y^2$ IV (MAGNITUDE PROPORTIONAL TO DISTANCE TO ORIGIN)

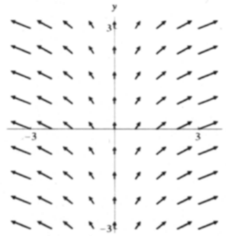
(d) $\vec{F} = \left\langle \frac{x}{\sqrt{x^2+y^2}}, \frac{y}{\sqrt{x^2+y^2}}\right\rangle$ II (ALL SAME MAGNITUDE)



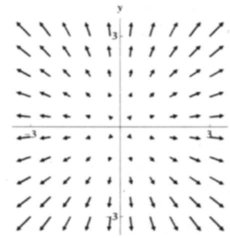
(I)



(II)



(III)



(IV)