

1. (8 points) Find the work done by the force field

$$\vec{F}(x, y, z) = \langle x - y^2, y - z^2, z - x^2 \rangle$$

on a particle that moves along the line segment from $(0, 0, 1)$ to $(2, 1, 0)$.

2. (8 points) Circle the vector fields below that are conservative throughout their domain.

(a) $\langle \sin x \cos y, \sin y \cos x \rangle$

(c) $\langle \ln(xy) + \frac{x}{y}, \ln(xy) + \frac{y}{x} \rangle, x > 0, y > 0$

(b) $\langle e^{x+y} + 2x + y, e^{x+y} + 2y + x \rangle$

(d) $\langle \ln(xy) + \frac{y}{x}, \ln(xy) + \frac{x}{y} \rangle, x > 0, y > 0$

3. Let

$$\vec{F}(x, y, z) = \langle \sin y, x \cos y + \cos z, -y \sin z + 1 \rangle, \text{ and}$$

$$\vec{r}(t) = \langle \sin t, t, 2t \rangle, \quad 0 \leq t \leq \pi/2.$$

(a) (4 points) Find a function $f(x, y, z)$ such that $\vec{F} = \nabla f$.

(b) (4 points) Use part (a) to evaluate $\int_C \vec{F} \cdot d\vec{r}$.

4. (8 points) Use Green's Theorem to evaluate the line integral

$$\oint_C y^3 dx - x^3 dy,$$

where C is the circle $x^2 + y^2 = 4$.

Bonus

5. (4 points (bonus)) Is the vector field given by

$$\vec{F}(x, y) = \left\langle \frac{-y}{x^2 + y^2}, \frac{x}{x^2 + y^2} \right\rangle$$

conservative throughout its domain? Explain briefly.