Name: *ANSWER KEY* 3/8/2017

Math 392 Linear Algebra and Vector Analysis for Engineers **Quiz 2**

1. (8 points) Find the work done by the force field

$$\vec{F}(x,y) = \langle x - y^2, y - z^2, z - x^2 \rangle$$

on a particle that moves along the line segment from $(0,0,1)$ to $(2,1,0)$.

$\vec{r}(t) = \langle 2t, t, 1-t \rangle, \quad 0 \leq t \leq 1$

$\vec{r}\,'(t) = \langle 2, 1, -1 \rangle$

$$W = \int_C \vec{F} \cdot d\vec{r} = \int_0^1 \vec{F}(\vec{r}(t)) \cdot \vec{r}\,'(t)\, dt$$

$$= \int_0^1 \langle 2t - t^2, t - (1-t)^2, 1 - t - 4t^2 \rangle \cdot \langle 2, 1, -1 \rangle \, dt$$

$$= \int_0^1 4t - 2t^2 + t - 1 + 2t - t^2 - 1 + t + 4t^2 \, dt$$

$$= \int_0^1 t^2 + 8t - 2 \, dt = \frac{1}{3}t^3 + 4t^2 - 2t \Big|_0^1$$

$$= \frac{1}{3} + 4 - 2 = \frac{1 + 12 - 6}{3} = \boxed{\frac{7}{3}}$$

2. (8 points) Circle the vector fields below that are conservative throughout their domain.

(a) $\langle \sin x \cos y, \sin y \cos x \rangle$

$P_y = -\sin x \sin y$

$Q_x = -\sin y \sin x$ ✓

(b) $\langle e^{x+y} + 2x + y, e^{x+y} + 2y + x \rangle$

$P_y = e^{x+y} + 1$

$Q_x = e^{x+y} + 1$ ✓

(c) $\langle \ln(xy) + \frac{x}{y}, \ln(xy) + \frac{y}{x} \rangle, x > 0, y > 0$

$P_y = \frac{1}{y} - \frac{x}{y^2}$

$Q_x = \frac{1}{x} - \frac{y}{x^2}$ NO

(d) $\langle \ln(xy) + \frac{y}{x}, \ln(xy) + \frac{x}{y} \rangle, x > 0, y > 0$

$P_y = \frac{1}{y} + \frac{1}{x}$

$Q_x = \frac{1}{x} + \frac{1}{y}$ ✓

3. Let

$$\vec{F}(x, y, z) = \langle \sin y, x \cos y + \cos z, -y \sin z + 1 \rangle, \text{ and}$$

$$\vec{r}(t) = \langle \sin t, t, 2t \rangle, \quad 0 \le t \le \pi/2.$$

(a) (4 points) Find a function $f(x, y, z)$ such that $\vec{F} = \nabla f$.

$f_x = \sin y \Rightarrow f = x \sin y + g(y,z)$

THEN $f_y = x \cos y + g_y(y,z) = x \cos y + \cos z$

$\Rightarrow g_y(y,z) = \cos(z) \Rightarrow g(y,z) = y\cos(z) + h(z)$

THEN $f = x \sin y + y \cos z + h(z)$

$\Rightarrow f_z = -y \sin z + h'(z) = -y \sin(z) + 1 \Rightarrow h'(z) = 1 \Rightarrow h(z) = z + C$

$\therefore \boxed{f(x,y,z) = x \sin y + y \cos z + z + C}$

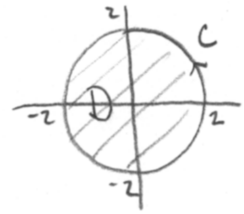
(b) (4 points) Use part (a) to evaluate $\int_C \vec{F} \cdot d\vec{r}$.

$\int_C \vec{F} \cdot d\vec{r} = \int_C \nabla f \cdot dr = f(\vec{r}(\frac{\pi}{2})) - f(\vec{r}(0))$

$= f(1, \frac{\pi}{2}, \pi) - f(0,0,0) = 1 \cdot \sin\frac{\pi}{2} + \frac{\pi}{2}\cos\pi + \pi - (0 + 0 + 0)$

$= 1 - \frac{\pi}{2} + \pi = \boxed{1 + \frac{\pi}{2}}$

4. (8 points) Use Green's Theorem to evaluate the line integral



$$\oint_C y^3\, dx - x^3\, dy,$$

where C is the circle $x^2 + y^2 = 4$.

$$\oint_C y^3 dx - x^3 dy = \iint_D \frac{\partial}{\partial x}[-x^3] - \frac{\partial}{\partial y}[y^3]\, dA$$

BY GREEN'S THM, SINCE BOTH P & Q HAVE CONTINUOUS PARTIAL DERIVATIVES THROUGHOT $\mathbb{R}^2$.

$$= -3 \iint_D x^2 + y^2\, dA$$

USE POLAR COORD: $x = r\cos\theta$, $y = r\sin\theta$, $dA = r\,dr\,d\theta$

$$= -3 \int_0^{2\pi}\int_0^2 r^3\, dr\, d\theta = -3 \int_0^{2\pi} 4\, d\theta = \boxed{-24\pi}$$

Bonus

5. (4 points (bonus)) Is the vector field given by

$$\vec{F}(x, y\text{[redacted]}) = \left\langle \frac{-y}{x^2 + y^2}, \frac{x}{x^2 + y^2} \right\rangle$$

conservative thoughout its domain? Explain briefly.

$\boxed{\text{No.}}$ ALTHOUGH IT IS TRUE THAT $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$,

$\boxed{\text{THE DOMAIN OF } \vec{F} \text{ IS } \mathbb{R}^2 \setminus \{(0,0)\}, \text{ WHICH IS } \underline{\text{NOT}} \text{ SIMPLY CONNECTED.}}$

(THE INTEGRAL AROUND A CLOSED CURVE THAT GOES AROUND THE ORIGIN WILL <u>NOT</u> BE 0.)