Name:____ **Due 5/15/2017**

Math 392 Linear Algebra and Vector Analysis for Engineers **Quiz 4**

1. Let A, B, C, D be the column matrices

$$A = \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}, \quad B = \begin{bmatrix} -7 \\ 1 \\ -8 \end{bmatrix}, \quad C = \begin{bmatrix} -5 \\ -1 \\ 6 \end{bmatrix}, \quad D = \begin{bmatrix} 3 \\ 1 \\ -1 \end{bmatrix}.$$

For each of the following, state whether the matrix multiplication(s) can be performed. If not, state why not. If so, evaluate the expression.

(a) (4 points) $A^T A$

(b) (4 points) $AB^T + CD^T$

(c) (4 points) $AD - BC$

2. Consider the following system of liner equations.

$$\begin{array}{rcrcrcl} -2x & + & y & - & z & = & 4 \\ x & + & 2y & + & 3z & = & 13 \\ 3x & & & + & z & = & 1 \end{array}$$

(a) (2 points) Give the augmented matrix that corresponds to this system.

(b) (4 points) Give the reduced-row echelon form of the matrix from part (a).

3. (8 points) Describe the general solution to the following system as a linear combination of column vectors.

$$\begin{array}{rcrcrcrcr} w & + & x & - & y & + & z & = & -1 \\ 2w & + & 3x & - & 3y & + & 2z & = & -4 \\ -w & - & 2x & + & 2y & - & z & = & 3 \end{array}$$

4. Let

$$A = \begin{bmatrix} 1 & 2 & -2 & 0 \\ 2 & 3 & -4 & 1 \\ -1 & -2 & 0 & 2 \\ 0 & 2 & 5 & 3 \end{bmatrix} \quad \text{and} \quad B = \frac{1}{5}A^2A^T$$

(a) (8 points) Find $\det A$.

(b) (4 points) Use the answer to part (a) to find $\det B$.

5. (8 points) Find the values of λ for which the following matrix has determinant 0.

$$\begin{bmatrix} 1-\lambda & 0 \\ 3 & -3-\lambda \end{bmatrix}$$

(Note: These are called the *eigenvalues* of the matrix $\begin{bmatrix} 1 & 0 \\ 3 & -3 \end{bmatrix}$.)

6. (8 points) Use Cramer's rule to solve the following system of equations.

$$\begin{array}{rcrcr} 2x & + & 3y & = & 2 \\ 4x & - & 9y & = & -1 \end{array}$$

7. Let E be the solid bounded below by $z = x^2 + y^2$ and above by $z = 4$, and let S be the boundary surface of E with outward pointing normal. Let $\vec{F}$ be the vector field

$$\vec{F} = x\vec{\imath} + y\vec{\jmath} + 1\vec{k}.$$

Calculate $\displaystyle\iint_S \vec{F} \cdot d\vec{S}$

(a) (8 points) directly as a surface integral, and

(b) (8 points) as a triple integral, by using the divergence theorem.