

Signed Numbers & Evaluating Expressions

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The Negative Sign

When a negative sign appears before a term (which may be a number, a letter, or a product of both numbers and letters) it means that the term is being multiplied by -1 . And when a term is multiplied by -1 , its sign is switched from positive to negative or from negative to positive. Thus, the term $-x$, read “negative x ”, is not necessarily negative; it is negative if x is positive, but it is positive if x is negative. (Similarly, “positive x ” is negative when x is negative. Thus,

$$-(-x) = (-1)((-1)x) = ((-1)(-1))x = 1x = x,$$

that is,

“A double negative yields a positive.”

Furthermore,

$$-xy = x(-y) = (-1)(xy),$$

“Positive times negative and negative times positive is negative;”

$$(-x)(-y) = (-1)x(-1)y = (-1)(-1)xy = 1xy = xy,$$

“Negative times negative and positive times positive is positive.”

In summary, the product of two factors with the same sign is positive, and the product of two factors with opposite signs is negative.

Furthermore, when dealing with products of many factors, the product is positive when there are an even number of negative factors, and the product is negative when there is an odd number of negative factors. And, of course, this applies to division as well. For example,

$$\frac{-a}{-b} = \frac{a}{b}, \quad \frac{a(-b)c}{(-a)b(-c)} = -1, \quad (-2)^4 = 16, \quad (-2)^5 = -32.$$

Remark. Unless the negative sign is inside of parentheses with an exponent, exponents do not act on the negative sign. Note the difference:

$$-a^n = -(a^n) \quad \text{and} \quad (-a)^n = \begin{cases} (-1)^n a^n = a^n & \text{if } n \text{ is even} \\ (-1)^n a^n = -(a^n) & \text{if } n \text{ is odd.} \end{cases}$$

Addition and Subtraction with Negative Numbers

It may help our discussion of negative numbers if we use the unit of dollars. Then negative numbers represent debt. And so it is clear that the sum of two negative numbers (adding debt to debt) is negative (more debt). This also makes clear that subtracting a negative number (taking away debt) is the same as adding a positive number (giving money).

$$a - (-b) = a + b$$

Remark. The distributive property yields the following.

$$a - b = -(b - a)$$

Thus,

$$13 - 48 = -(48 - 13) = -35 \quad \text{and} \quad \frac{a - b}{b - a} = -1.$$

Practice p. 207-208

Evaluating Expressions

There are two important things to keep in mind:

1. Parentheses are just one way of grouping terms together and forcing these expressions to be evaluated first. Brackets and bars (whether in fractions or roots) perform the exact same role.
2. When plugging in numbers for letters, parentheses should be placed around the numbers so that any exponents are applied to the negative sign as well, and so negative signs are not confused with subtraction signs.

Practice: p. 213, 215 (grouping)

Practice: p. 217, 219 (plugging in)