

5.1 Simple and Compound Interest

Simple Interest

Generally used for investments/loans lasting less than one year.

Example 1. A Payday loan company has offered you a short term loan of \$1,500 at 83.2% annual interest. How much do you owe in interest, and how much do you owe in total if you pay the loan back ...

- (a) 3 months later?
- (b) 2 weeks later?
- (c) 5 days later?

Simple Interest

$$I = Prt$$

where

- P is the principal;
- r is the annual interest rate;
- t is the time in years.

Future or Maturity Value for Simple Interest

The **future** or **maturity value** A of P dollars at a simple interest rate r for t years is

$$A = P(1 + rt).$$

Compound Interest

Generally used for investments/loans lasting more than one year.

Example 2. Dr. Z has accumulated \$14,250 in credit card debt on a card that charges 26.4% annual interest. Ignoring any late fees, if she does not make any payments and does not make any further charges, what is the amount owed 2 years later if interest is compounded ...

- (a) annually?
- (b) monthly?

Compound Amount

$$A = P(1 + i)^n$$

where $i = \frac{r}{m}$ and $n = mt$,

A is the future (maturity) value;

P is the principal;

r is the annual interest rate;

m is the number of compounding periods per year;

t is the number of years;

n is the number of compounding periods;

i is the interest rate per period.

Example 3. On the day of their daughter's birth, two new parents invest x dollars in US Treasury bonds that earn 2.5% annual interest. When their daughter turns 18, the bonds are worth \$10,000. Find x if interest is compounded ...

(a) semi-annually.

(b) monthly.

Present Value for Compound Interest

The **present value** of A dollars compounded at an interest rate i per period for n periods is

$$P = \frac{A}{(1 + i)^n} \quad \text{or} \quad P = A(1 + i)^{-n}.$$

Example 4. Suppose \$100 is invested in an account that earns 100% annual interest. What is the account balance after 1 year if interest is compounded ...

(a) annually?

(b) quarterly?

(c) daily?

In each case, what is the effective rate of interest?

Effective Rate

The **effective rate** corresponding to a stated rate of interest r compounded m times per year is

$$r_E = \left(1 + \frac{r}{m}\right)^m - 1.$$

Doubling Time

Example 5. A contestant on the TV show “Shark Tank” projects that her company’s valuation will increase by 4% every year. At this rate, approximately how long will it take for a shark’s investment to double? What if her company’s valuation increases by 8% every year?

Annual Interest Rate r	Approximate Doubling Time (years)
$.001 \leq r < .05$	$\frac{70}{100r}$
$.05 \leq r \leq .12$	$\frac{72}{100r}$