

5.3 Present Value of an Annuity, Amortization

Future vs Present Value of an Annuity

Example 1. Dr. A and Dr. Z go to the same investment firm and open identical accounts that earn 3.6% annual interest compounded monthly.

- Dr. A makes periodic payments of R dollars at the end of every month.
- Dr. Z makes one deposit of P dollars immediately and no further deposits.

After 5 years, both accounts have a balance of \$10,000. Find R and P .

Remark 1. We can use future value as a way of equating these different payment structures: money up front vs periodic payments. Voila! Structured loans like credit cards and mortgages!

Present Value of an Ordinary Annuity

The present value P of an annuity of n payments of R dollars each at the end of consecutive interest periods with interest compounded at a rate of interest i per period is

$$P = R \left[\frac{1 - (1 + i)^{-n}}{i} \right] \quad \text{or} \quad P = Ra_{\overline{n}|i}.$$

Example 2. Wendy has \$4,200 of debt on a credit card that charges 28.8% annual interest, and she wants to pay off her debt in 14 equal monthly payments of R dollars. Find R .

Amortization Payments

A loan of P dollars at interest rate i per period may be amortized in n equal periodic payments of R dollars made at the end of each period, where

$$R = \frac{P}{\left[\frac{1 - (1 + i)^{-n}}{i} \right]} = \frac{Pi}{1 - (1 + i)^{-n}} \quad \text{or} \quad R = \frac{P}{a_{\overline{n}|i}}.$$

Ammortization Tables

Example 3. *You've been approved for a \$300,000 thirty year mortgage that charges 8% annual interest compounded monthly.*

- (a) *What are your monthly payments?*
- (b) *How much of your first payment goes toward paying off interest, and how much goes toward paying off the principal?*
- (c) *How much of your 100th payment goes toward paying off interest, and how much goes toward paying off the principal?*
- (d) *How much of your 359th payment goes toward paying off interest, and how much goes toward paying off the principal?*

You may use this [Ammortization Table](#)