EXAM 2 : MON 11/16 , IN CLASS "PAPER" EXAM

**Fordham Math 1108, Math for Business: Finite** **Practice Problems for Exam 2**

1. A restaurant serves 12 side dishes – 3 potato dishes, 5 vegetable dishes, and 4 pasta dishes. Customers are allowed to shoose three distinct side dishes.
   (a) How many possible side dish combinations can one order at this restaurant?
   (b) How many possible side dish combinations can one order at this restaurant if you have to order 1 potato dish, 1 vegetable dish, and 1 pasta dish?
   (c) How many possible side dish combinations can one order at this restaurant if you have to order exactly two vegetable dishes?
   (d) How many possible side dish combinations can one order at this restaurant if you have to order at least two vegetable dishes?
2. A club with 22 members must select a president, a vice-president, and secretary from among themselves. How many ways can they do this?
3. A club with 25 members – 17 women and 8 men – must select 5 members to attend a club fair. If they want to send 3 women and 2 men, how many possible ways can they do this?
4. Let
$$A = \{a, b, c, d, e, f\}, \qquad B = \{a, e, i, o, u\}$$
   (a) Find $A \cap B$.
   (b) Find $A \cup B$.
   (c) List all subsets of $A \cap B$.
   (d) How many subset of $B$ exist?
   (e) If the universal set $U$ is the 26-letter alphabet, how many elements are in $A' \cap B'$?
5. A parking lot contains 150 cars. 35 cars are red, 65 cars are SUVs, and 75 cars are neither red no SUVs. How many cars are red SUVs?
6. A family has two children.
   (a) What is the probability that both children were born on the weekend?
   (b) Given that neither child was born on a Monday, what is the probability that both children were born on the weekend?
   (c) Are the events "both children were born on the weekend" and "neither child was born on a Monday" inedependent events?
   (d) Are the events "both children were born on the weekend" and "neither child was born on a Monday" mutually exclusive events?
7. When the weather is dry, the probability that your flight will be delayed is 10%. When is it raining, the probabiity that your flight will be delayed is 25%. When it is snowing, the probability that your flight will be delayed is 45%. Suppose the probability of rain is 18% and the probability of snow is 13%.
   (a) What is the probability that your flight will be delayed?
   (b) Suppose you are woken up by an alert that your flight is delayed, before you have a chance to check the weather. What is the probability that is snowing?
8. Calculate the following.
$$\sum_{k=2}^{6} \frac{5k+1}{2^k - 1}$$
9. A random sample of 6 bullfrogs were studied in their natural habitat, and the number of times that they croaked over a period of 15 minutes was recorded. This data is listed below.
$$35, 19, 26, 52, 26, 34$$
   Find the mean, median, mode, and standard deviation for the set of data.

Module 5: counting (sections 7.3 and 7.4)

- product rule
- permutations
- Combinations

1. A restaurant serves 12 side dishes – 3 potato dishes, 5 vegetable dishes, and 4 pasta dishes. Customers are allowed to shoose three distinct side dishes.
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Product rule: if an event can broken into k stages, and the first stage has n1 possible outcomes, the second stage has n2 possible, outcomes, kth stage has nk possible outcomes, then there are a total of

$$n_1 \times n_2 \times n_3 \times \dots \times n_k \quad \text{POSSIBLE OUTCOMES.}$$

(a) $_{12}C_3 = \frac{12!}{3!(12-3)!} = \frac{\cancel{12}^{2} \cdot 11 \cdot 10 \cdot \cancel{9!}}{\cancel{3} \cdot \cancel{2} \cdot 1 \cdot \cancel{9!}} = 220$ (COMBINATIONS)

(b) $\underbrace{3}_{\text{POT}} \times \underbrace{5}_{\text{VEG}} \times \underbrace{4}_{\text{PASTA}} = 60$ (PRODUCT RULE)

(c) $\underbrace{_5C_2}_{\text{VEG.}} \times \underbrace{_7C_1}_{\text{NON-VEG}} = 10 \times 7 = 70$

$$\frac{n!}{0!\,n!} = {}_nC_0 = 1 = {}_nC_n \quad (0! = 1)$$

(d) # VEG. DISHES $\geq 2$ =

$$\underbrace{\underbrace{_5C_2}_{\text{VEG.}} \times \underbrace{_7C_1}_{\text{NON-VEG}}}_{\text{2 VEG. DISHES}} + \underbrace{\underbrace{_5C_3}_{\text{VEG.}} \times \underbrace{_7C_0}_{\text{NON-VEG.}}}_{\text{3 VEG. DISHES}} = 70 + 10 = 80$$

$$\frac{5!}{3!\,2!}$$

$$_nC_r = {}_nC_{n-r}$$

$$\frac{n!}{r!(n-r)!} = \frac{n!}{(n-r)!(n-(n-r))!} = \frac{n!}{(n-r)!r!} \quad \checkmark$$

2. A club with 22 members must select a president, a vice-president, and secretary from among themselves. How many ways can they do this?

PERMUTATIONS: $_{22}P_3 = \frac{22!}{(22-3)!} = 22 \times 21 \times 20$

$$\underset{\text{PREZ}}{\underline{22}} \times \underset{\text{V.P.}}{\underline{21}} \times \underset{\text{SEC.}}{\underline{20}} = 9240$$

DIFFERENT POSITIONS $\Rightarrow$ ORDER MATTERS

3. A club with 25 members – 17 women and 8 men – must select 5 members to attend a club fair. If they want to send 3 women and 2 men, how many possible ways can they do this?

ORDER CHOSEN DOESN'T MATTER

$$\underset{\text{WOMEN}}{\underline{_{17}C_3}} \times \underset{\text{MEN}}{\underline{_8C_2}} = 680 \times 28 = 19{,}040$$

## Module 6: Sets and Probability (sections 7.2, 8.1, 8.2, 8.3)

- ◯ Sets, elements, subsets, empty set, notation, set builder notation
- ◯ Intersection, union, complement, "mutually exclusive"
- ◯ Addition rule
- ◯ Venn diagram
- ◯ Listing/visualizing simple events (all equally likely)
- ◯ Calculating probabilities using P(A)=n(A)/n(S)

4. Let

$$A = \{a, b, c, d, e, f\}, \qquad B = \{a, e, i, o, u\}$$

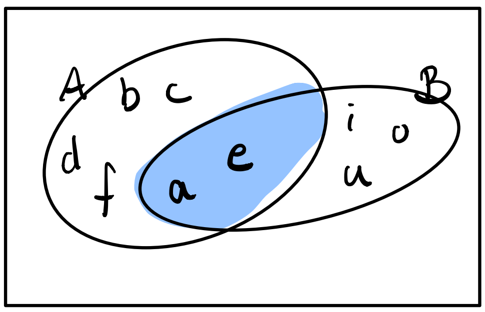
(a) Find $A \cap B$.
(b) Find $A \cup B$.
(c) List all subsets of $A \cap B$.
(d) How many subset of $B$ exist?
(e) If the universal set $U$ is the 26-letter alphabet, how many elements are in $A' \cap B'$?

SET IS COLLECTION OF OBJECTS CALLED ELEMENTS

$\{ \_, \_, \_ \}$

SET

ELEMENTS.




"AND"

(a) $\cap$ INTERSECTION $\rightarrow$ $A \cap B = \{a, e\}$

NOTE: $\{a, e\} \subseteq A$, $\{a, e\} \subseteq B$ (SUBSET $\subseteq$)

(b) $\cup$ UNION $\rightarrow$ $A \cup B = \{a, b, c, d, e, f, i, o, u\}$

"OR"

(INCLUSIVE BOTH OK)

(c) $A \cap B = \{a, e\}$ ← LIST ALL POSSIBLE SUBSETS.

GO THROUGH ELEMENTS ONE-BY-ONE → $\underset{\text{INCLUDE } a?}{\underline{Y/N}}$ $\underset{\text{INCLUDE } e?}{\underline{Y/N}}$

| DECISIONS | | SUBSETS |
|---|---|---|
| Y Y | → | $\{a, e\}$ |
| Y N | → | $\{a\}$ |
| N Y | → | $\{e\}$ |
| N N | → | $\{\ \} = \emptyset$ EMPTY SET |

$\emptyset \subseteq S$ FOR ALL SETS $S$

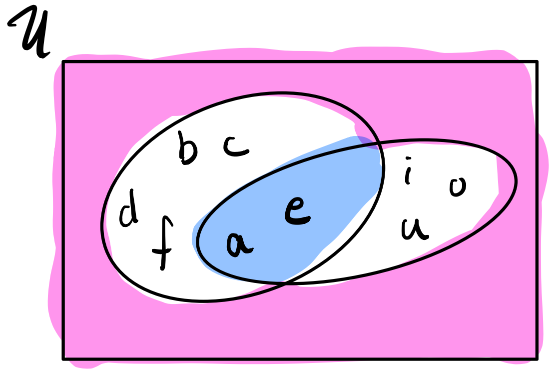
(d) $\underset{a}{\underline{Y/N}}$ $\underset{e}{\underline{Y/N}}$ $\underset{i}{\underline{Y/N}}$ $\underset{o}{\underline{Y/N}}$ $\underset{u}{\underline{Y/N}}$

$$2 \times 2 \times 2 \times 2 \times 2 = 2^5 = 32$$

NOTATION: $n(A) = \#$ ELEMENTS IN $A$,

$$\#\text{ SUBSETS OF } A = 2^{n(A)}$$

(e)

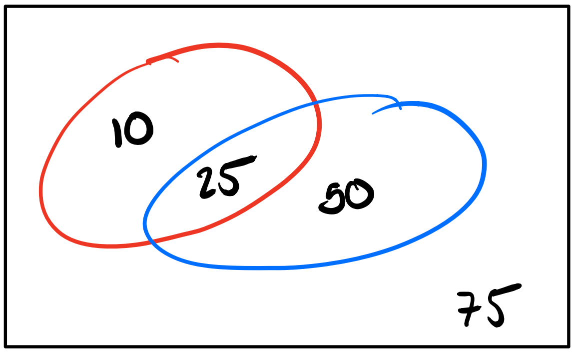



$A' \cap B'$ = NOT A AND NOT B

$= \{g, h, j, k, l, m, n, p, q, r, s, t, v, w, x, y, z\}$

nor

5. A parking lot contains 150 cars. 35 cars are red, 65 cars are SUVs, and 75 cars are neither red no SUVs. How many cars are red SUVs?




RED

SUVs

ADDITION RULE:

$$n(R \cup S) = n(R) + n(S) - n(R \cap S)$$

$$75 = 35 + 65 - n(R \cap S)$$

$$n(R \cap S) = 35 + 65 - 75 = 25$$

6. A family has two children.
   (a) What is the probability that both children were born on the weekend?
   (b) Given that neither child was born on a Monday, what is the probability that both children were born on the weekend?
   (c) Are the events “both children were born on the weekend” and ”neither child was born on a Monday” inedependent events?
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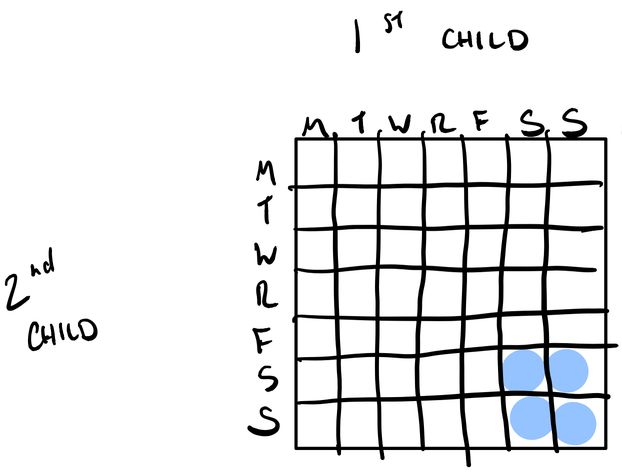


SAMPLE SPACE $S$: EACH □ REPRESENTS A POSSIBLE OUTCOME

$$n(S) = 49$$

LET $A$ = BOTH BORN ON WEEKEND

$$P(A) = \frac{n(A)}{n(S)} = \frac{4}{49}$$

BECAUSE ALL SIMPLE EVENTS ARE EQUALLY LIKELY.

Module 7: Conditional probability (sections 8.3, 8.4)

- ◯ Definition of conditional probability
- ◯ Multiplication rule
- ◯ Definition of independent events
- ◯ Bayes' formula

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A \cap B) = P(B)P(A|B)$$

6. A family has two children.
   (a) What is the probability that both children were born on the weekend?
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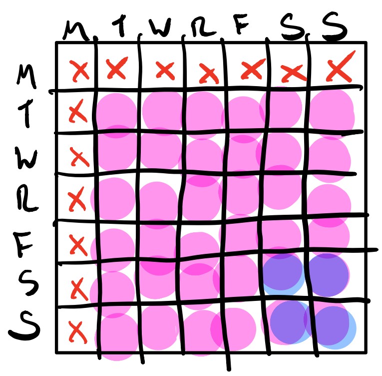
(b)




LET $A$ = BOTH BORN ON WEEKEND

NOW PROB OF $A = \frac{4}{36}$

↑ REMOVED OUTCOMES NO LONGER POSSIBLE

LET $B$ = NEITHER BORN ON MONDAY

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{4/49}{36/49} = \frac{4}{36}$$

(c) A & B ARE INDEPENDENT IF ANY OF THE FOLLOWING ARE TRUE:

$$P(A|B) = P(A)$$

$$P(B|A) = P(B)$$

$$P(A \cap B) = P(A)P(B|A) = P(A)P(B)$$

ALWAYS

ONLY IF A,B INDEPENDENT

$P(A) = \frac{4}{49}$ , $P(A|B) = \frac{4}{36}$

So No, A & B ARE NOT INDEPENDENT.

(d) A & B ARE MUTUALLY EXCLUSIVE IF

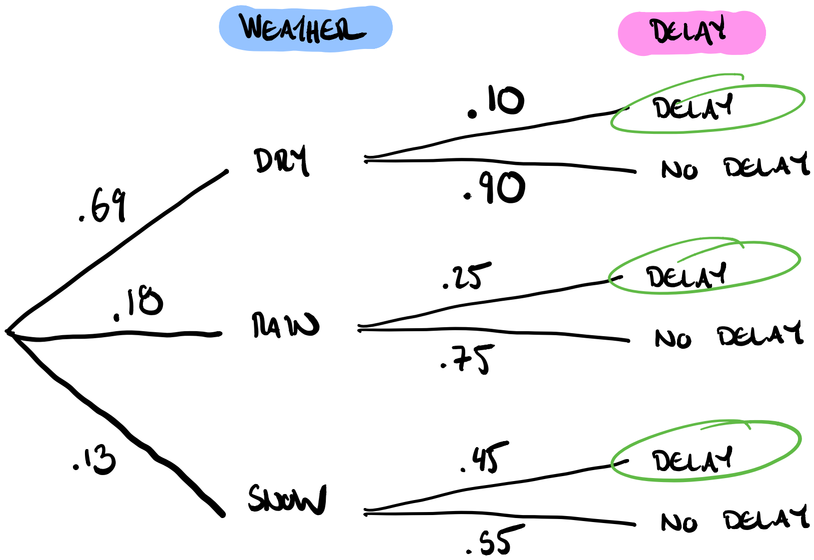
$P(A \cap B) = 0$ (OR IF $A \cap B = \emptyset$ )

No, $P(A \cap B) = \frac{4}{49} \neq 0$.

7. When the weather is dry, the probability that your flight will be delayed is 10%. When is it raining, the probabiity that your flight will be delayed is 25%. When it is snowing, the probability that your flight will be delayed is 45%. Suppose the probability of rain is 18% and the probability of snow is 13%.

   (a) What is the probability that your flight will be delayed?

   (b) Suppose you are woken up by an alert that your flight is delayed, before you have a chance to check the weather. What is the probability that is snowing?




GIVEN CONDITIONAL PROB.

$P(\text{DELAY} \mid \text{DRY}) = .10$

$P(\text{DELAY} \mid \text{RAIN}) = .25$

$P(\text{DELAY} \mid \text{SNOW}) = .45$

(a)
$$\begin{aligned}
P(\text{DELAY}) &= P(\text{DELAY} \cap \text{DRY}) + P(\text{DELAY} \cap \text{RAIN}) + P(\text{DELAY} \cap \text{SNOW}) \\
&= P(\text{DRY})P(\text{DELAY} \mid \text{DRY}) + P(\text{RAIN})P(\text{DELAY} \mid \text{RAIN}) + P(\text{SNOW})P(\text{DELAY} \mid \text{SNOW}) \\
&= (.69)(.10) + (.18)(.25) + (.13)(.45) \\
&= .1725
\end{aligned}$$

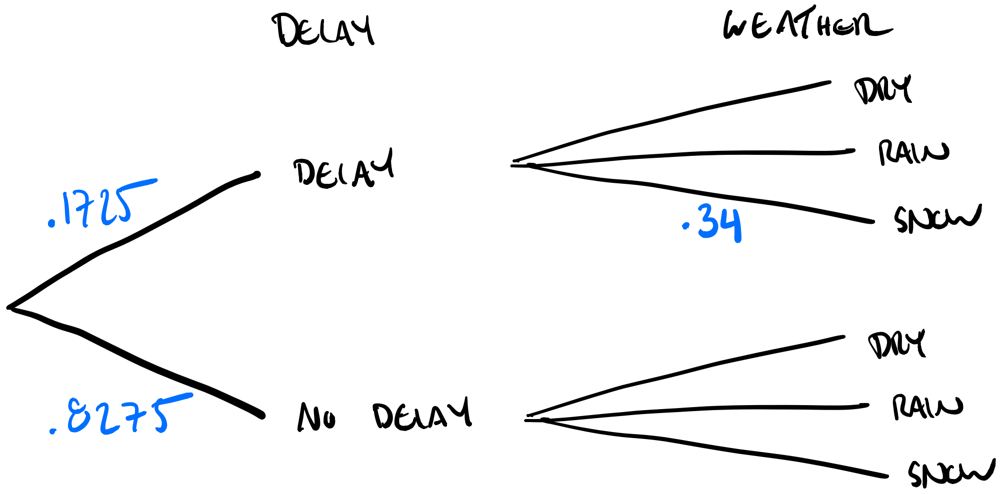
THIS IS THE RELATION!

(b) BAYES' FORMULA:
$$P(A \mid B) = \frac{P(A)P(B \mid A)}{P(B)}$$

$$P(\text{SNOW} \mid \text{DELAY}) = \frac{P(\text{SNOW})P(\text{DELAY} \mid \text{SNOW})}{P(\text{DELAY})}$$

$$= \frac{(.13)(.45)}{.1725} = .3391 \approx 34\%$$

PART (a)


DELAY
WEATHER
DELAY
NO DELAY
.1725
.8275
DRY
RAIN
SNOW
.34
DRY
RAIN
SNOW

Module 8: Descriptive statistics (sections 10.1, 10.2, 10.3)

- ◯ Frequency table, histogram, pie chart READ
- ◯ Sigma notation
- ◯ Mean, median, mode
- ◯ Standard deviation

END

8. Calculate the following.

$$\sum_{k=2}^{6} \frac{5k+1}{2^k - 1}$$

$$= \frac{5(2)+1}{2^2-1} + \frac{5(3)+1}{2^3-1} + \frac{5(4)+1}{2^4-1} + \frac{5(5)+1}{2^5-1} + \frac{5(6)+1}{2^6-1} = \ldots$$

9. A random sample of 6 bullfrogs were studied in their natural habitat, and the number of times that they croaked over a period of 15 minutes was recorded. This data is listed below.

$$35, 19, 26, 52, 26, 34$$

Find the mean, median, mode, and standard deviation for the set of data.

MEAN $\bar{x} = \dfrac{\sum x_i}{6} = \dfrac{35+19+26+\ldots+34}{6} = \underline{\underline{32}}$

MEDIAN $19 \quad 26 \quad \underbrace{26 \quad 34} \quad 35 \quad 52$

$\text{MEDIAN} = \dfrac{26+34}{2} = \underline{\underline{30}}$

MODE $= 26$

SAMPLE STANDARD DEV. $s = \sqrt{\dfrac{1}{n-1} \sum (x_i - \bar{x})^2}$

POPULATION STANDARD DEV $\sigma = \sqrt{\dfrac{1}{N} \sum (x_i - \mu)^2}$

| $x_i$ | $x_i - 32$<br>$x_i - \bar{x}$ | $(x_i - \bar{x})^2$ |
|---|---|---|
| 35 | 3 | 9 |
| 19 | -13 | 169 |
| 26 | -6 | 36 |
| 52 | 20 | 400 |
| 26 | -6 | 36 |
| 34 | 2 | 4 |

$s = \sqrt{\dfrac{1}{6-1}(9+169+36+400+36+4)}$

$s = \sqrt{130.8}$