FINAL EXAM Thnisoay 12/17 9:30-11:30 am (moles 1-10)

## 1 Standardization of Normal Distributions


$z$-scone: $\begin{array}{llllllll}z: & -3 & -2 & -1 & 0 & 1 & 2 & 3\end{array}$

$$
z=\frac{x-\mu}{\sigma}
$$

Figure 1: Standardizing a measurement $x$ is simply converting it to the "standard" until of number of standard deviations above ( + ) or below ( - ) the mean.

1. Suppose that you must establish regulations concerning the maximum number of people who can occupy an elevator. A study indicates that if eight people occupy the elevator, the probability distribution of the total weight of the eight people is approximately normally distributed with a mean equal to 1200 pounds and a standard deviation of 99 pounds. What is the probability that the total weight of eight people exceeds 1300 pounds? 1500 pounds?


## Appendix



## Area under the Standard Normal Curve



Area under the Standard Normal Curve

| (Table Entries Represent the Area under the Standard Normal Curve from 0 to $z, z \geq 0$ ) |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $z$ | . 00 | . 01 | . 02 | . 03 | . 04 | . 05 | . 06 | . 07 | . 08 | . 09 |
| 0.0 | 0.0000 | 0.0040 | 0.0080 | 0.0120 | 0.0160 | 0.0199 | 0.0239 | 0.0279 | 0.0319 | 0.0359 |
| 0.1 | 0.0398 | 0.0438 | 0.0478 | 0.0517 | 0.0557 | 0.0596 | 0.0636 | 0.0675 | 0.0714 | 0.0753 |
| 0.2 | 0.0793 | 0.0832 | 0.0871 | 0.0910 | 0.0948 | 0.0987 | 0.1026 | 0.1064 | 0.1103 | 0.1141 |
| 0.3 | 0.1179 | 0.1217 | 0.1255 | 0.1293 | 0.1331 | 0.1368 | 0.1406 | 0.1443 | 0.1480 | 0.1517 |
| 0.4 | 0.1554 | 0.1591 | 0.1628 | 0.1664 | 0.1700 | 0.1736 | 0.1772 | 0.1808 | 0.1844 | 0.1879 |
| 0.5 | 0.1915 | 0.1950 | 0.1985 | 0.2019 | 0.2054 | 0.2088 | 0.2123 | 0.2157 | 0.2190 | 0.2224 |
| 0.6 | 0.2257 | 0.2291 | 0.2324 | 0.2357 | 0.2389 | 0.2422 | 0.2454 | 0.2486 | 0.2517 | 0.2549 |
| 0.7 | 0.2580 | 0.2611 | 0.2642 | 0.2673 | 0.2704 | 0.2734 | 0.2764 | 0.2794 | 0.2823 | 0.2852 |
| 0.8 | 0.2881 | 0.2910 | 0.2939 | 0.2967 | 0.2995 | 0.3023 | 0.3051 | 0.3078 | 0.3106 | 0.3133 |
| 0.9 | 0.3159 |  | 0.3212 | 0.3238 |  | 0.3289 | 0.3315 | 0.3340 | 0.3365 | 0.3389 |
| 1.0 | 0.3413 | 0.3438 | 0.3461 | 0.3485 | 0.3508 | 0.3531 | 0.3554 | 0.3577 | 0.3599 | 0.3621 |
| 1.1 | 0.3643 |  | 0.3686 | 0.3708 | T | 0.3749 | 0.3770 | 0.3790 | 0.3810 | 0.3830 |
| 1.2 | 0.3849 | 0.3869 | 0.3888 | 0.3907 | 0.3925 | 0.3944 | 0.3962 | 0.3980 | 0.3997 | 0.4015 |
| 1.3 | 0.4032 | 0.4049 | 0.4066 | 0.4082 | 0.4099 | 0.4115 | 0.4131 | 0.4147 | 0.4162 | 0.4177 |
| 1.4 | 0.4192 | 0.4207 | 0.4222 | 0.4236 | 0.4251 | 0.4265 | 0.4279 | 0.4292 | 0.4306 | 0.4319 |
| 1.5 | 0.4332 | 0.4345 | 0.4357 | 0.4370 | 0.4382 | 0.4394 | 0.4406 | 0.4418 | 0.4429 | 0.4441 |
| 1.6 | 0.4452 | 0.4463 | 0.4474 | 0.4484 | 0.4495 | 0.4505 | $0-15$ | 0.4525 | 0.4535 | 0.4545 |
| 1.7 | 0.4554 | 0.4564 | 0.4573 | 0.4582 | 0.4591 | 0.4599 | 0.4608 | 0.4616 |  | 0.4633 |
| 1.8 | 0.4641 | 0.4649 | 0.4656 | 0.4664 | 0.4671 | 0.4678 | 0.7006 | 0.4693 | 0.4699 | 0.4706 |
| 1.9 | 0.4713 | 0.4719 | 0.4726 | 0.4732 | 0.4738 | 0.4744 | 0.4750 | 0.4756 | 0.4701 | 0.4767 |
| 2.0 | 0.4772 | 0.4778 | 0.4783 | 0.4788 | 0.4793 | 0.4798 | 0.4803 | 0.4808 | 0.4812 | 0.4817 |
| 2.1 | 0.4821 | 0.4826 | 0.4830 | 0.4834 | 0.4838 | 0.4842 | 0.4846 | 0.4850 | 0.4854 | 0.4857 |
| 2.2 | 0.4861 | 0.4864 | 0.4868 | 0.4871 | 0.4875 | 0.4878 | 0.4881 | 0.4884 | 0.4887 | 0.4890 |
| 2.3 | 0.4893 | 0.4896 | 0.4898 | 0.4901 | 0.4904 | 0.4906 | 0.4909 | 0.4911 | 0.4913 | 0.4916 |
| 2.4 | 0.4918 | 0.4920 | 0.4922 | 0.4925 | 0.4927 | 0.4929 | 0.4931 | 0.4932 | 0.4934 | 0.4936 |
| 2.5 | 0.4938 | 0.4940 | 0.4941 | 0.4943 | 0.4945 | 0.4946 | 0.4948 | 0.4949 | 0.4951 | 0.4952 |
| 2.6 | 0.4953 | 0.4955 | 0.4956 | 0.4957 | 0.4959 | 0.4960 | 0.4961 | 0.4962 | 0.4963 | 0.4964 |
| 2.7 | 0.4965 | 0.4966 | 0.4967 | 0.4968 | 0.4969 | 0.4970 | 0.4971 | 0.4972 | 0.4973 | 0.4974 |
| 2.8 | 0.4974 | 0.4975 | 0.4976 | 0.4977 | 0.4977 | 0.4978 | 0.4979 | 0.4979 | 0.4980 | 0.4981 |
| 2.9 | 0.4981 | 0.4982 | 0.4982 | 0.4983 | 0.4984 | 0.4984 | 0.4985 | 0.4985 | 0.4986 | 0.4986 |
| 3.0 | 0.4987 | 0.4987 | 0.4987 | 0.4988 | 0.4988 | 0.4989 | 0.4989 | 0.4989 | 0.4990 | 0.4990 |
| 3.1 | 0.4990 | 0.4991 | 0.4991 | 0.4991 | 0.4992 | 0.4992 | 0.4992 | 0.4992 | 0.4993 | 0.4993 |
| 3.2 | 0.4993 | 0.4993 | 0.4994 | 0.4994 | 0.4994 | 0.4994 | 0.4994 | 0.4995 | 0.4995 | 0.4995 |
| 3.3 | 0.4995 | 0.4995 | 0.4995 | 0.4996 | 0.4996 | 0.4996 | 0.4996 | 0.4996 | 0.4996 | 0.4997 |
| 3.4 | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4998 |
| 3.5 | 0.4998 | 0.4998 | 0.4998 | 0.4998 | 0.4998 | 0.4998 | 0.4998 | 0.4998 | 0.4998 | 0.4998 |
| 3.6 | 0.4998 | 0.4998 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 |
| 3.7 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 |
| 3.8 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 |
| 3.9 | 0.5000 | 0.5000 | 0.5000 | 0.5000 | 0.5000 | 0.5000 | 0.5000 | 0.5000 | 0.5000 | 0.5000 |
|  |  |  |  |  |  |  |  |  |  | 595 |

2. The flourescent light tubes made by the company Well-lit have lifetimes which are normally distributed with mean 2010 hours and standard deviation 20 hours. The company decides to promote its sales of the tubes by guaranteeing a minimum life of the tubes, replacing free of charge any tubes that fail to meet this minimum life. If the company wish to have to replace only $3 \%$ of the tubes sold, find the guaranteed minimum it must set.


FIND $x_{0}$ SUCH THAT $P\left(x<x_{0}\right)=.03$



REVERSE -LOOK-up. 47
in body of the table
$-z_{0}=1.88$

$z_{0}=-1.88$

https://youtu.be/ mbnBYh-BJIg

Figure 2: This is a pangolin.
3. The weight $x$ of an adult female pangolin is normally distriuted. If $15 \%$ of adult female pangolins weight less than 14 lbs and $3 \%$ of adult female pangolins weight more than 26 lbs , find the mean and standard deviation for the weight of an adult female pangolin.


2 ELis
2 UNKNOWNS

$$
-1.04=\frac{14-\mu}{\sigma}
$$

$$
1.8 \delta=\frac{26-\mu}{\sigma}
$$

save For $\mu, \sigma$



2 Normal Approximation to Binomial Distributions
Recall that when $x$ is the number of success in $n$ identical and independent Bernoulli trials each with probability of success $p$ and probability of failure $q=1-p$, the mean and standard deviation for $x$ are

$$
\mu=n p, \quad \sigma=\sqrt{n p q}
$$

As $n$ gets larger, the binomial distribution of $x$ gets closer to a normal distribution. The normal distribution it approaches has the same mean $\mu=n p$ and standard deviation $\sigma=\sqrt{n p q}$ as long as the following inequalities are satisfied.

$$
0<\mu-3 \sigma, \quad \mu+3 \sigma<n
$$

Equaivalently,

$$
0<n p-3 \sqrt{n p q}, \quad n p+3 \sqrt{n p q}<n
$$

These inequalities are a type of "rule of thumb" test.
4. Let $x$ be the number of successes in 80 identical and independent Bernoulli trials each with probability of success .65. Use a normal distribution to approximate the following probabilities.
(a) $P(x<45)$
(b) $P(x \leq 45)$
(c) $P(45<x<56)$
(d) $P(45 \leq x \leq 56)$
(a)

$$
p(x=0)+p(x=1)+\ldots+p(x=44)
$$

$$
=\sum_{i=0}^{141} P(x=i)=\sum_{i=0}^{44} C_{i}^{6}(.65)^{i}(.35)^{6-i}
$$


(binomial) \# successes in
$n$ Tills

$$
\left.\begin{array}{l}
P(x \times 45) \\
P(x \leq 44)
\end{array}\right\}=P\left(x_{\text {span }} \leq 44.5\right)
$$

$$
\begin{aligned}
& z=\frac{x-\mu}{\sigma}=\frac{x-n p}{\sqrt{n p q}} \\
& z=\frac{44.5-(80)(.65)}{\sqrt{(80)(.65)(.35)}}=-1.76
\end{aligned}
$$

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5. Airlines and hotels often grant reservations in excess of capacity to minimize losses due to no-shows. Suppose the records of a hotel show that, on the average, $10 \%$ of their prospective guests will not claim their reservation. If the hotel accepts 215 reservations and there are only 200 rooms in the hotel, what is the probability that all guests who arrive to claim a room will receive one?
6. Depending on the values of $p$ and $q$, how many repeated trials $n$ are necessary to obtain a binomial distribution that passes the rule-of-thumb test for using a normal distribution as a suitable approximation?
$p \dot{q} q$ are fixeD.
$n$ must SAISFY:
sole far $n$ :


