10.2 Measures of Central Tendency

Sigma Notation

Definition 1. Sigma notation is a convenient way of writing long sums of terms.

$$\sum_{i=1}^{4}(2^i + 3i) = (2^1 + 3(1)) + (2^2 + 3(2)) + (2^3 + 3(3)) + (2^4 + 3(4))$$
$$= 5 + 10 + 17 + 28$$
$$\sum_{i=1}^{n} x_i = x_1 + x_2 + x_3 + \ldots + x_n$$

Mean

DEFINITION Mean: Ungrouped Data

If $x_1, x_2, \ldots, x_n$ is a set of n measurements, then the **mean** of the set of measurements is given by

$$[\text{mean}] = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{x_1 + x_2 + \cdots + x_n}{n} \quad (1)$$

where

$$\bar{x} = [\text{mean}] \text{ if data set is a sample}$$
$$\mu = [\text{mean}] \text{ if data set is the population}$$

Example 1. Find the mean of the following set of measurements.

5 7 11 15 13 10 8 19

Example 2. The mean of 4 numbers is 90. If the mean of the first three numbers is 88, find the fourth number.

Example 3. Suppose I buy 20 gallons of gas at a price of $3.40/gallon, and you buy 10 gallons of gas at a price of $3.10/gallon. Together, what is the average price of gas per gallon that we've paid?

Example 4. Find the mean for the sample data summarized in the table below.

Class Interval	Frequency
14.5-19.5	2
19.5-24.5	4
24.5-29.5	5
29.5-34.5	6
34.5-39.5	3
39.5-44.5	1

DEFINITION Mean: Grouped Data

A data set of n measurements is grouped into k classes in a frequency table. If x_i is the midpoint of the ith class interval and f_i is the ith class frequency, then the **mean for the grouped data** is given by

$$[\text{mean}] = \frac{\sum_{i=1}^{k} x_i f_i}{n} = \frac{x_1 f_1 + x_2 f_2 + \cdots + x_k f_k}{n} \tag{2}$$

where

$$n = \sum_{i=1}^{k} f_i = \text{total number of measurements}$$

$$\bar{x} = [\text{mean}] \text{ if data set is a sample}$$

$$\mu = [\text{mean}] \text{ if data set is the population}$$

Median

Example 5. The following two sets of data give the salaries of the 6 employees at a small startup company before and after the CEO is given a raise.

1. 42,000 64,000 64,000 82,000 82,000 96,000
2. 42,000 64,000 64,000 82,000 82,000 650,000

Calculate the mean, median, and mode for each set of data.

Example 6. Consider the following set of measurements.

16 22 26 28 40 46 x

Depending on x, what are the possible values of the median?

DEFINITION Median: Ungrouped Data

1. If the number of measurements in a set is odd, the **median** is the middle measurement when the measurements are arranged in ascending or descending order.
2. If the number of measurements in a set is even, the **median** is the mean of the two middle measurements when the measurements are arranged in ascending or descending order.

Shapes of distributions

Mean μ compared to median m	Distribution
$\mu < m$	left-skewed
$\mu = m$	symmetric
$\mu > m$	right-skewed

Mode

DEFINITION Mode

The **mode** is the most frequently occurring measurement in a data set. There may be a unique mode, several modes, or, if no measurement occurs more than once, essentially no mode.

Example 7. Find the mode(s) for each of the following sets of data.

A:	69	54	59	52	53	66	70	52	70	60
B:	60	60	57	69	52	56	52	50	54	54
C:	61	57	70	84	52	62	80	71	81	89