

Bernoulli Trials and Binomial Distributions

Definition 1. A *Bernoulli trial* is any experiment that has only two possible outcomes. By convention, we refer to these two outcomes abstractly as success and failure, and we denote the probability of success p and the probability of failure $q = 1 - p$.

Example 1. Consider the Bernoulli trial of spinning a spinner such that the probability of spinning red (success) is .4 and the probability of spinning white (failure) is .6. Suppose you repeat this Bernoulli trial $n = 5$ times.

1. What is the probability that the outcome is $WWRRW$?
2. What is the probability that the total number of successes $x = 2$?

THEOREM 1 Probability of x Successes in n Bernoulli Trials

The probability of exactly x successes in n independent repeated Bernoulli trials, with the probability of success of each trial p (and of failure q), is

$$P(x \text{ successes}) = {}_n C_x p^x q^{n-x} \quad (1)$$

Definition 2. A sequence of n independent Bernoulli trials is called a Binomial experiment. When we define the random variable x to be the number of successes in a *binomial experiment*, it's called a *binomial random variable* and its probability distribution is called a *binomial distribution*.

Example 2. Marbles from a jar with/without replacement. Binomial experiment?

Example 3. Consider the binomial experiment of spinning the spinner from example 1 $n = 5$ times. Define the random variable x to be the number of successes. Describe the probability distribution for x by filling in the following table.

x	
$p(x)$	

Remark 1. We can visualize distributions for binomial random variables with histograms.
<https://www.stat.berkeley.edu/stark/Java/Html/BinHist.htm>

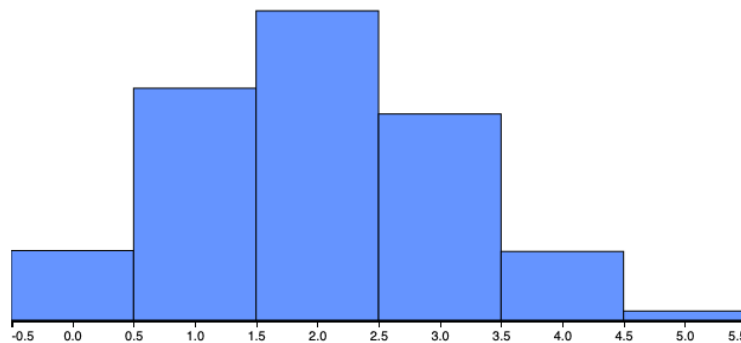
Theorem 1. Consider a binomial experiment composed of n independent Bernoulli trials, each with probability of success p and probability of failure q . Define the random variable x to be the number of successes observed. Then the expected value/mean and standard deviation for x are

$$\mu = np$$

and

$$\sigma = \sqrt{npq}$$

Example 4. Find the mean and standard deviation for the binomial random variable x from example 3.



Example 5. Suppose every time a basketball player takes a free throw, the probability that she makes the shot is 80%. If she takes 6 free throws in one game, what is the probability that she makes exactly 5? At least 5?

Example 6. Imagine two different six-sided fair dice, called die A and die B .

- Die A has its faces labeled 1, 1, 1, 2, 2, 3.
- Die B has its faces labeled 1, 2, 2, 3, 3, 3.

What is the probability that both dice are rolled simultaneously 10 times and doubles are rolled exactly 4 times?

S	1	1	1	2	2	3
1						
2						
2						
3						
3						
3						

Example 7. A printer is broken in such a way that every time a page is printed, there is a 30% chance that that printed page will contain a defect. If 20 pages are printed, what is the probability that more than 1 page contains a defect? What is the probability that less than 5 pages contain a defect?

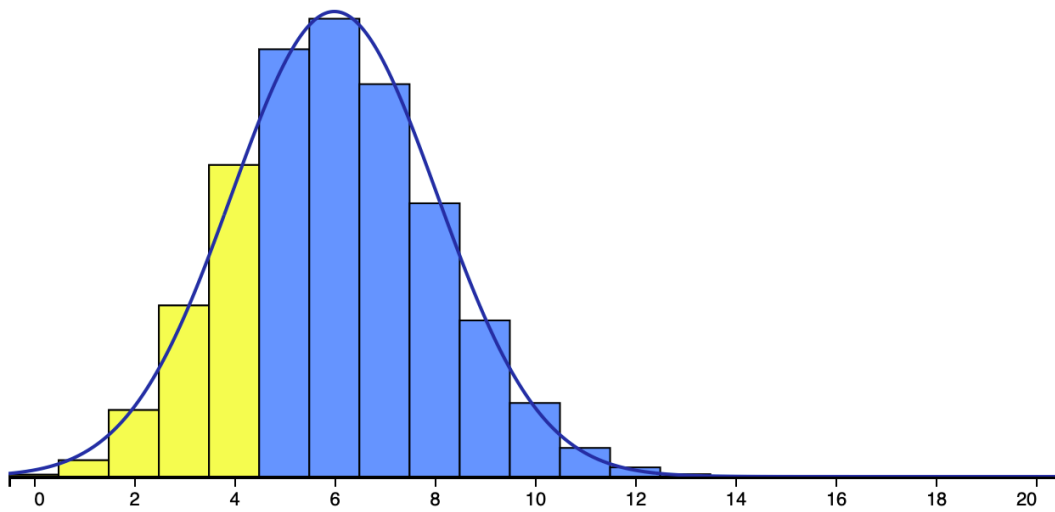


Figure 1: <https://www.stat.berkeley.edu/stark/Java/Html/BinHist.htm>