18. Maximize $P = 30x + 12y$
subject to $3x + y \le 18$
$x, y \ge 0$

19. Minimize $C = 8x + 9y$
subject to $5x + 6y \ge 60$
$x, y \ge 0$

MAXIMIZATION / MINIMIZATION PROBLEM

↳ OPTIMIZE THE OBJECTIVE FUNCTION SUBJECT TO CONSTRAINTS (FEASIBLE REGION)

30. Minimize and maximize
$P = 2x + y$
subject to
$x + y \ge 2$
$6x + 4y \le 36$
$4x + 2y \le 20$
$x, y \ge 0$

GRAPH SEVERAL LEVEL CURVES, ALL SAME SLOPE

https://www.desmos.com/calculator/oja5lhphbs

THE OPTIMAL SOLUTION IS THE POINT(S) THAT LIE
(1) IN THE FEASIBLE REGION
(2) ON THE LEVEL CURVE WITH THE $\underbrace{\text{LARGEST/SMALLEST VALUE FOR THE OBJECTIVE FUNCTION}}_{\text{OPTIMAL VALUE}}$.

THIS OCCURS AT CORNER POINTS
INTERSECTIONS OF BOUNDARY COMPONENTS.

THEOREM 1 Fundamental Theorem of Linear Programming

If the optimal value of the objective function in a linear programming problem exists, then that value must occur at one or more of the corner points of the feasible region.

THEOREM 2 Existence of Optimal Solutions

(A) If the feasible region for a linear programming problem is bounded, then both the maximum value and the minimum value of the objective function always exist.
(B) If the feasible region is unbounded and the coefficients of the objective function are positive, then the minimum value of the objective function exists but the maximum value does not.
(C) If the feasible region is empty (that is, there are no points that satisfy all the constraints), then both the maximum value and the minimum value of the objective function do not exist.

51. Production scheduling. A furniture company has two plants that produce the lumber used in manufacturing tables and chairs. In 1 day of operation, plant A can produce the lumber required to manufacture 20 tables and 60 chairs, and plant B can produce the lumber required to manufacture 25 tables and 50 chairs. The company needs enough lumber to manufacture at least 200 tables and 500 chairs.

(A) If it costs $1,000 to operate plant A for 1 day and $900 to operate plant B for 1 day, how many days should each plant be operated to produce a sufficient amount of lumber at a minimum cost? What is the minimum cost?

(B) Discuss the effect on the operating schedule and the minimum cost if the daily cost of operating plant A is reduced to $600 and all other data in part (A) remain the same.

(C) Discuss the effect on the operating schedule and the minimum cost if the daily cost of operating plant B is reduced to $800 and all other data in part (A) remain the same.

PROCEDURE Geometric Method for Solving a Linear Programming Problem with Two Decision Variables

Step 1 Graph the feasible region. Then, if an optimal solution exists according to Theorem 2, find the coordinates of each corner point.
Step 2 Construct a **corner point table** listing the value of the objective function at each corner point.
Step 3 Determine the optimal solution(s) from the table in Step 2.
Step 4 For an applied problem, interpret the optimal solution(s) in terms of the original problem.

58. Capital expansion. A fast-food chain plans to expand by opening several new restaurants. The chain operates two types of restaurants, drive-through and full-service. A drive-through restaurant costs $100,000 to construct, requires 5 employees, and has an expected annual revenue of $200,000 A full-service restaurant costs $150,000 to construct, requires 15 employees, and has an expected annual revenue of $500,000. The chain has $2,400,000 in capital available for expansion. Labor contracts require that they hire no more than 210 employees, and licensing restrictions require that they open no more than 20 new restaurants. How many restaurants of each type should the chain open in order to maximize the expected revenue? What is the maximum expected revenue? How much of their capital will they use and how many employees will they hire?

PROCEDURE Constructing a Model for an Applied Linear Programming Problem

Step 1 Introduce decision variables.
Step 2 Summarize relevant material in table form, relating columns to the decision variables, if possible (see Table 1).
Step 3 Determine the objective and write a linear objective function.
Step 4 Write problem constraints using linear equations and/or inequalities.
Step 5 Write nonnegative constraints.