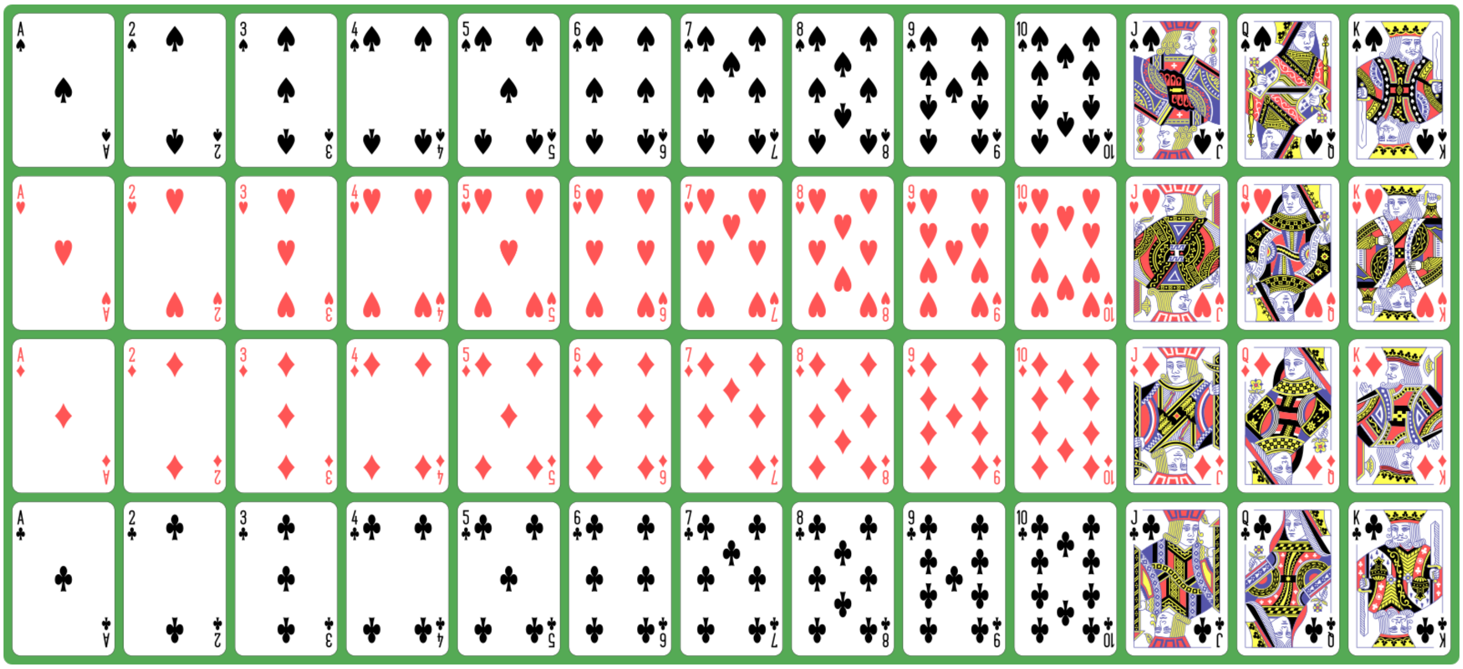
§8.1 Sample Spaces, Events, & Probability

Def: An experiment is any procedure by which an observation is made.
A single repetition of an experiment is called a trial.

e.g. Tossing a coin
Rolling a die
Selecting a card from a shuffled deck

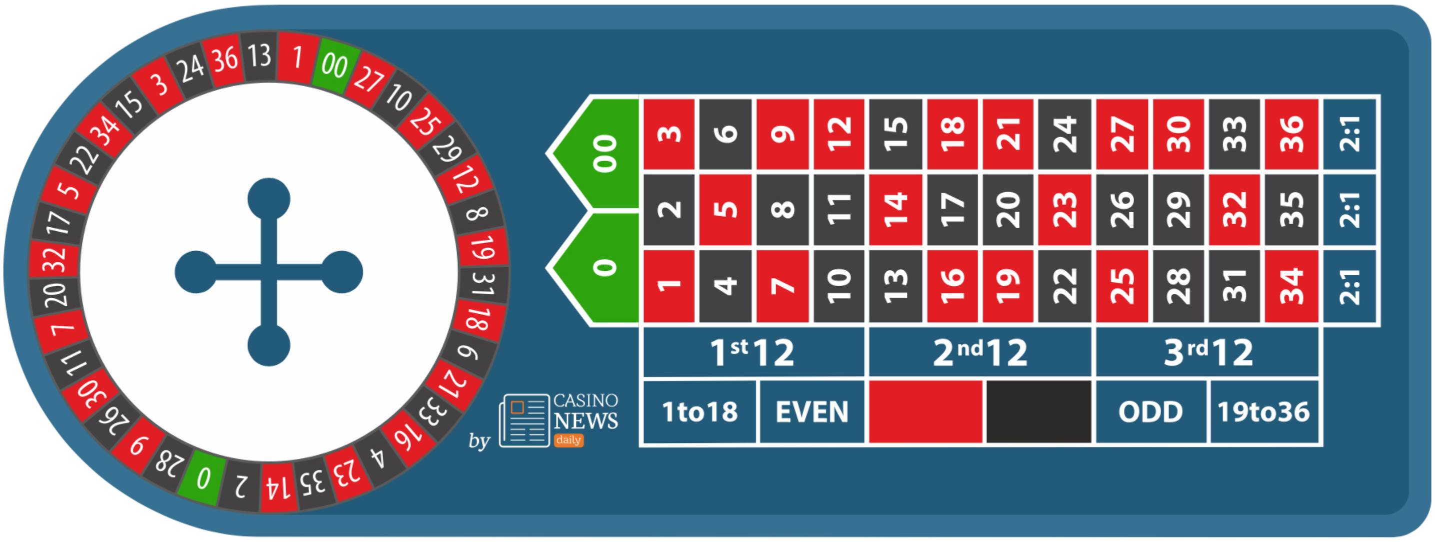
Def: Given an experiment, a sample space S is a set of outcomes such that each trial results in one & only one outcome.



ex. Experiment: choose one card from a shuffled standard deck of 52 cards.
Give several different sample spaces for this experiment.

Def: Any subset of the sample space S is called an event.
An event that consists of only one element from the sample space is caled a simple event, and events that consist of 2 or more are called compound events.

Given an event $E \subset S$, we say that the event E occurs if any of the simple events in E occurs.



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Sample Spaces and Events Consider an experiment of rolling two dice. Figure 2 shows a convenient sample space that will enable us to answer many questions about interesting events. Let S be the set of all ordered pairs in the figure. The simple event (3, 2) is distinguished from the simple event (2, 3). The former indicates that a 3 turned up on the first die and a 2 on the second, while the latter indicates that a 2 turned up on the first die and a 3 on the second.

Second Die

First Die	1	2	3	4	5	6
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

Figure 2

What is the event (subset of the sample space S) that corresponds to each of the following outcomes?

(A) A sum of 7 turns up.

(B) A sum of 11 turns up.

(C) A sum less than 4 turns up.

(D) A sum of 12 turns up.

DEFINITION Probabilities for Simple Events

Given a sample space

$$S = \{e_1, e_2, \ldots, e_n\}$$

with n simple events, to each simple event e_i we assign a real number, denoted by $P(e_i)$, called the **probability of the event e_i**. These numbers can be assigned in an arbitrary manner as long as the following two conditions are satisfied:

Condition 1. The probability of a simple event is a number between 0 and 1, inclusive. That is,

$$0 \leq P(e_i) \leq 1$$

Condition 2. The sum of the probabilities of all simple events in the sample space is 1. That is,

$$P(e_1) + P(e_2) + \cdots + P(e_n) = 1$$

Any probability assignment that satisfies Conditions 1 and 2 is said to be an **acceptable probability assignment**.

DEFINITION Probability of an Event *E*

Given an acceptable probability assignment for the simple events in a sample space S, we define the **probability of an arbitrary event E**, denoted by $P(E)$, as follows:

(A) If E is the empty set, then $P(E) = 0$.
(B) If E is a simple event, then $P(E)$ has already been assigned.
(C) If E is a compound event, then $P(E)$ is the sum of the probabilities of all the simple events in E.
(D) If E is the sample space S, then $P(E) = P(S) = 1$ [this is a special case of part (C)].

ex. LET $S = \{e_1, e_2, e_3, e_4, e_5\}$

AND $P(e_1) = .11$ $P(e_2) = .16$ $P(e_3) = .32$ $P(e_4) = .25$ $P(e_5) = x$.

(a) FIND x.
(b) IF $E = \{e_1, e_2, e_5\}$, FIND $P(E)$.
(c) FIND $P(E')$.

EXAMPLE 4 **Probabilities of Events** Let us return to Example 2, the tossing of a nickel and a dime, and the sample space

$$S = \{\text{HH}, \text{HT}, \text{TH}, \text{TT}\}$$

Since there are 4 simple outcomes and the coins are assumed to be fair, it would appear that each outcome would occur 25% of the time, in the long run. Let us assign the same probability of $\frac{1}{4}$ to each simple event in S:

Simple Event				
e_i	**HH**	**HT**	**TH**	**TT**
$P(e_i)$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

This is an acceptable assignment according to Conditions 1 and 2, and it is a reasonable assignment for ideal (perfectly balanced) coins or coins close to ideal.

(A) What is the probability of getting 1 head (and 1 tail)?

(B) What is the probability of getting at least 1 head?

(C) What is the probability of getting at least 1 head or at least 1 tail?

(D) What is the probability of getting 3 heads?

Matched Problem 4 Suppose in Example 4 that after flipping the nickel and dime 1,000 times, we find that HH turns up 273 times, HT turns up 206 times, TH turns up 312 times, and TT turns up 209 times. On the basis of this evidence, we assign probabilities to the simple events in S as follows:

Simple Event				
e_i	**HH**	**HT**	**TH**	**TT**
$P(e_i)$	.273	.206	.312	.209

This is an acceptable and reasonable probability assignment for the simple events in S. What are the probabilities of the following events?

(A) E_1 = getting at least 1 tail

(B) E_2 = getting 2 tails

(C) E_3 = getting at least 1 head or at least 1 tail

1. *Theoretical Approach.* We use assumptions and a deductive reasoning process to assign probabilities to simple events. No experiments are actually conducted. This is what we did in Example 4.
2. *Empirical Approach.* We assign probabilities to simple events based on the results of actual experiments. This is what we did in Matched Problem 4.

ex. AN EXPERIMENT CONSISTS OF SELECTING ONE CARD FROM A SHUFFLED DECK OF 52 CARDS.

(a) FIND THE PROBABILITY OF SELECTING AN ACE.

(b) FIND THE PROBABILITY OF SELECTING A FACE CARD.

(c) FIND THE PROBABILITY OF SELECTING A SPADE.

THEOREM 1 Probability of an Arbitrary Event under an Equally Likely Assumption

If we assume that each simple event in sample space S is as likely to occur as any other, then the probability of an arbitrary event E in S is given by

$$P(E) = \frac{\text{number of elements in } E}{\text{number of elements in } S} = \frac{n(E)}{n(S)}$$

In Problems 79–86, find the probability of being dealt the given hand from a standard 52-card deck. Refer to the description of a standard 52-card deck on page 384.

79. A 5-card hand that consists entirely of red cards

80. A 5-card hand that consists entirely of face cards

81. A 6-card hand that contains exactly two face cards

82. A 6-card hand that contains exactly two clubs

83. A 4-card hand that contains no aces

84. A 4-card hand that contains no face cards

85. A 7-card hand that contains exactly 2 diamonds and exactly 2 spades

86. A 7-card hand that contains exactly 1 king and exactly 2 jacks

Example. An experiment consists of selecting 5 cards from a shuffled deck of 52 cards.

1. Find the probability of selecting five spades.
2. Find the probability of selecting 4-of-a-kind.
3. Find the probability of selecting a full house.
4. Find the probability of selecting two pairs.
5. Find the probability of selecting a straight.

Example. If you flip a coin 20 times, what is the probability that you flip exactly 8 heads?