Random Variables, Probability Distributions and Expected Value

DEFINITION Random Variable

A **random variable** is a function that assigns a numerical value to each simple event in a sample space S.

Sample Space S	Number of Heads $X(e_i)$
e_1: TTT	0
e_2: TTH	1
e_3: THT	1
e_4: HTT	1
e_5: THH	2
e_6: HTH	2
e_7: HHT	2
e_8: HHH	3

*The probability distribution p of the random variable X is defined by $p(x) = P(\{e_i \in S | X(e_i) = x\})$, which, because of its cumbersome nature, is usually simplified to $p(x) = P(X = x)$ or simply $p(x)$. We will use the simplified notation.

Number of Heads x	0	1	2	3
Probability $p(x)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

THEOREM 1 Probability Distribution of a Random Variable X

The **probability distribution of a random variable** X, denoted by $P(X = x) = p(x)$, satisfies

1. $0 \le p(x) \le 1, \quad x \in \{x_1, x_2, \ldots, x_n\}$
2. $p(x_1) + p(x_2) + \cdots + p(x_n) = 1$

where $\{x_1, x_2, \ldots, x_n\}$ are the (range) values of X (see Fig. 2).

DEFINITION Expected Value of a Random Variable X

Given the probability distribution for the random variable X,

x_i	x_1	x_2	...	x_n
p_i	p_1	p_2	...	p_n

where $p_i = p(x_i)$, we define the **expected value of X**, denoted $E(X)$, by the formula

$$E(X) = x_1p_1 + x_2p_2 + \cdots + x_np_n$$

Example. On your niece's birthday, you play a game with her. You put three \$1 bills, two \$20 bills, and one \$100 bill into a box, shake it up, and she removes one bill that she gets to keep. Let x equal the amount of money won by your neice.

1. Describe the proability distribution for x by filling in the following table.

x	
$p(x)$	

2. Find the expected value $E(x)$.

Example. Previous problem, but change the game so your neice removes two bills (without replacement).

Example. From experience, a shipping company knows that the cost of delivering a small package is \$12. The company charges \$16 for shipment but guarantees to refund the charge if delivery is not made within 24 hours. Suppose the company fails to deliver 2% of its packages within the 24-hour delivery period. Let x equal the profit that the company gains/loses by delivering a single package.

1. Describe the proability distribution for x by filling in the following table.

x	
$p(x)$	

2. Find the expected value $E(x)$.

Example. A school sells 1,500 raffle tickets for 10 each. Five tickets are chosen to receive a small prize of \$100, 3 ticket are chosen to receive a runer-up prize of \$500, and 1 ticket is chosen to receive a grand prize of \$5,000. Let x be the amount of money won/lost (positive/negative) by purchasing a single raffle ticket.

1. Describe the proability distribution for x by filling in the following table.

x	
$p(x)$	

2. Find the expected value $E(x)$.

Example (bonus). A labor union for coal miners wants to offer a one-year disability insurance policy to its members. The union wants to give $500,000 to any policy holder that experiences a disabling work-related injury. If the probability that a miner experiences a disabling work-related injury in any one-year period is 0.8%, how much should the union charge for a one-year disability insurance policy in order to expect to beak even?

Example. Four AA batteries are randomly selected from a drawer that contains 16 AA batteries, 9 of which are new and 7 of which are dead. Let x equal the number of fresh batteries selected.

1. Describe the proability distribution for x by filling in the following table.

x	
$p(x)$	

2. Find the expected value $E(x)$.