Name: * ANSWER KEY * 10/15/2020

Math 1108-L01, Math for Business: Finite Exam 1

1. (8 points) Your friend reluctantly agrees to loan you \$20 so you can buy a metro card. However, your friend is not very charitable, and is going to charge you an annual simple interest rate of 800%. If you wait three weeks before paying your friend back, how much do you owe them? Assume 1 year = 52 weeks, and round your answer to the nearest penny.

$$\boxed{A = P + I = P(1 + rt)}$$

$P = 20$

$r = 8$

$t = \frac{3}{52}$

$A = (20)(1 + 8 \cdot \frac{3}{52})$

$A = (20)(\frac{76}{52})$

$A = 29.2307\ldots \approx \boxed{\$29.23}$

2. (10 points) Suppose you purchase from the government a 20 year bond that earns 1.06% annual interest, compounded semiannually. If the bond has a face value of \$1000 (that is, the value of the bond will be \$1000 after 20 years), what is the purchase price of the bond?

$$\boxed{A = P\left(1 + \frac{r}{n}\right)^{nt} = P\left(1 + r_E\right)^t}$$

$\Rightarrow P = \frac{A}{(1 + \frac{r}{n})^{nt}}$

$A = 1000$

$r = .0106$

$n = 2$

$t = 20$

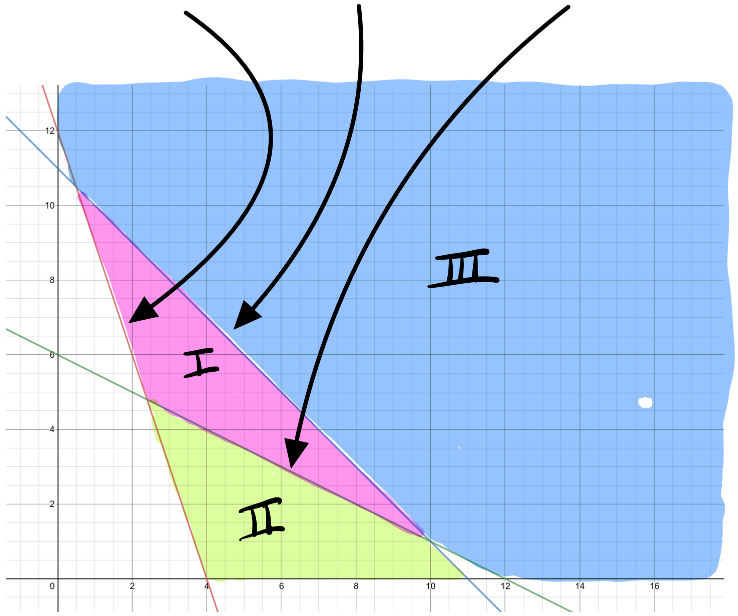
$P = \frac{1000}{(1 + \frac{.0106}{2})^{(2)(20)}}$

$P = \frac{1000}{1.0053^{40}} = 809.4177\ldots$

$P = \boxed{\$809.42}$

3. The graphs of the following three *equations* are shown below.

$$6x + 2y = 24, \qquad x + y = 11, \qquad x + 2y = 12$$



(a) (8 points) Write a system of *inequalities* such that the solution region is the shaded region labelled I.

$6x + 2y \geq 24$

$x + y \leq 11$

$x + 2y \geq 12$

(b) (8 points) Write a system of *inequalities* such that the solution region is the shaded region labelled II.

$6x + 2y \geq 24$ $\qquad y \geq 0$

$x + y \leq 11$

$x + 2y \leq 12$

(c) (8 points) Why is every linear objective function of x and y (e.g. $z = ax + by$) guarenteed to attain both a maximum and minimum value over the shaded regions labelled I and II, but not III?

REGIONS I & II ARE BOUNDED

REGION III IS UNBOUNDED

4. (10 points) Suppose a young adult accepts a student loan of $10,000 that charges 9% annual interest, compounded monthly. If payments of $100 are made at the end of each month, how many months will it take to pay off the loan? Round up to the next whole number of months.

$$\boxed{P = \frac{R[1-(1+i)^{-n}]}{i}}$$

$$\Rightarrow \quad Pi = R\left[1-(1+i)^{-n}\right]$$

SOLVE FOR n

$P = 10{,}000$

$R = 100$

$i = \frac{.09}{12} = .0075$

$$\frac{Pi}{R} = 1-(1+i)^{-n}$$

$$(1+i)^{-n} = 1 - \frac{Pi}{R}$$

$$-n = \log_{(1+i)}\left(1-\frac{Pi}{R}\right)$$

$$n = -\log_{(1+i)}\left(1-\frac{Pi}{R}\right) = \frac{-\ln\left(1-\frac{Pi}{R}\right)}{\ln(1+i)}$$

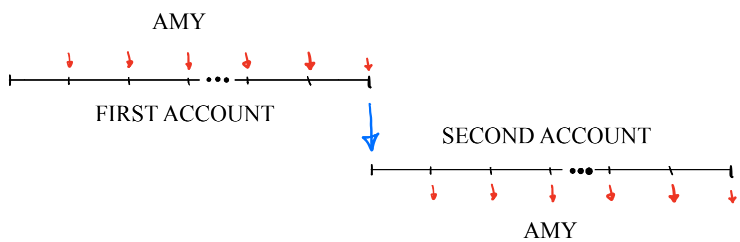
PLUG IN:

$$n = \frac{-\ln\left(1-\frac{(10{,}000)(.0075)}{100}\right)}{\ln(1.0075)}$$

$$n = \frac{-\ln(.25)}{\ln(1.0075)} = 185.5315\ldots$$

$$n \approx \boxed{186 \text{ MONTHS}}$$

5. Every month for 20 years Amy deposits \$250 into an account that earns 2% annual interest, compounded monthly. Just after making the final deposit, Amy withdraws all of the money in the account. She then takes this large amount of money and deposits it into a second account earning 3% annual interest, compounded monthly. At the end of every month for the next 20 years, Amy makes an equal size withdrawal from the second account such that after the last withdrawal the balance in the second account is \$0.



(a) (10 points) How much does Amy withdraw from the first account (and deposit into the second account)?

(b) (10 points) How much does Amy withdraw from the second account each month?

(c) (8 points) What is the total amount of interest that Amy earns from both accounts?

(a) $\boxed{S_n = \frac{R[(1+i)^n - 1]}{i}}$ $\Rightarrow$ $S_n = \dfrac{250\left[\left(1+\frac{.02}{12}\right)^{240} - 1\right]}{\frac{.02}{12}}$

$R = 250$

$i = \frac{.02}{12}$

$n = 12 \times 20 = 240$

$\approx \boxed{\$73{,}699.21}$

(b) $\boxed{P = \frac{R[1-(1+i)^{-n}]}{i}}$ $\Rightarrow$ $R = \dfrac{Pi}{1-(1+i)^{-n}}$

$P = 73{,}699.21$

$i = \frac{.03}{12} = .0025$

$n = 12 \times 20 = 240$

$= \dfrac{(73{,}699.21)(.0025)}{1 - 1.0025^{-240}}$

$\approx \boxed{\$408.73}$

(c) TOTAL OUT − TOTAL IN $= (240)(408.73) - (240)(250)$

$= 240\,(408.73 - 250) = \boxed{\$38{,}095.20}$

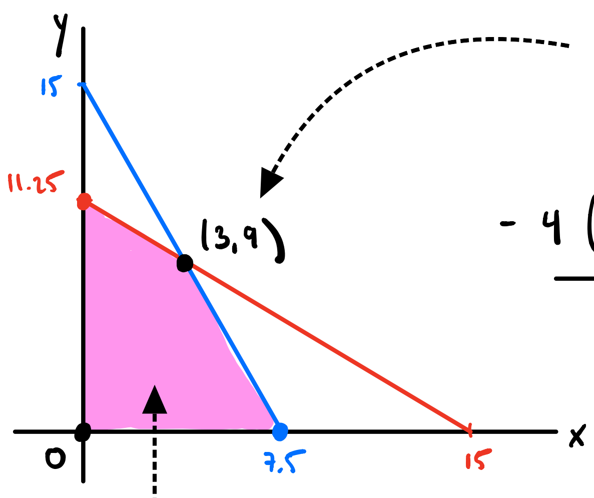
6. (20 points) A pottery hobbyist has decided to open a small online store to sell handmade ceramic mugs and bowls. Each mug requires 3 ounces of clay, takes 2 hours to make, and sells for \$22. Each bowl requires 4 ounces of clay, takes 1 hour to make, and sells for \$18. If she can only use 45 ounces of clay each week, and can only devote 15 hours per week to pottery, how many mugs and how many bowls should she make each week to maximize her sales?

	OUNCES OF CLAY	HOURS TO MAKE	SELLS FOR
MUG	3	2	22
BOWL	4	1	18

LET x = # MUGS PER WEEK, y = # BOWLS PER WEEK, z = SALES PER WEEK

RESTRICTIONS: $3x + 4y \leq 45$, $x \geq 0$, $2x + y \leq 15$, $y \geq 0$.

MAXIMIZE THE OBJECTIVE FUNCTION $z = 22x + 18y$



INTERSECTION:

$$3x + 4y = 45$$
$$-4\,(2x + y = 15)$$
$$-5x = -15$$
$$x = 3 \Rightarrow y = 9$$

Since the solution region is bounded, the objective function attains a maximum (and minimum) value. Since the objective function and all inequalities are linear, the maximum (and minimum) value is attained at a corner point of the solution region.

POINT (x,y)	SALES $z = 22x + 18y$
(0, 11.25)	202.5
(3, 9)	228
(7.5, 0)	165
(0, 0)	0

3 MUGS, 9 BOWLS YIELDS MAX SALES OF \$228

Financial Math Formulas

Simple interest

- I = interest
- P = principal
- r = annual interest rate (decimal)
- t = time (years)
- A = account balance/future value

$$\boxed{I = Prt} \qquad \boxed{A = P + I = P(1 + rt)}$$

Compound interest

- P = principal
- r = annual interest rate (decimal)
- n = number of compound periods per year
- t = time (years)
- A = account balance/compound amount
- r_E = effective rate/annual percentage yield (APY)

$$\boxed{A = P\left(1 + \frac{r}{n}\right)^{nt} = P\left(1 + r_E\right)^t} \qquad \boxed{r_E = \left(1 + \frac{r}{n}\right)^n - 1}$$

Future value of an annuity

- S_n = future value of annuity
- n = total number of payment periods
- R = recurring payment
- i = interest rate per payment/compound period
- D_n = future value of annuity due

$$\boxed{S_n = \frac{R[(1+i)^n - 1]}{i}} \qquad \boxed{D_n = S_{n+1} - R}$$

Present value of an annuity

- P = present value of annuity
- n = total number of payment periods
- R = recurring payment
- i = interest rate per payment/compound period

$$\boxed{P = \frac{R[1 - (1+i)^{-n}]}{i}}$$