Exam 1

Answer all 9 questions for a total of 100 points. Write your solutions in the accompanying blue book, and put a box around your final answers. If you solve the problems out of order, please skip pages so that your solutions stay in order. Good luck!

Financial Math Formulas

$$I = Prt, \qquad A = (1+i)^n P$$

$$FV = PMT \cdot \frac{(1+i)^n - 1}{i}, \qquad PV = PMT \cdot \frac{(1+i)^{-n} - 1}{i}$$

1. (10 points) Graph the solution region of the system of linear inequalities. Label the coordinates of any/all x-intercepts, y-intercepts, and corner points. State whether the solution region is bounded or unbounded.

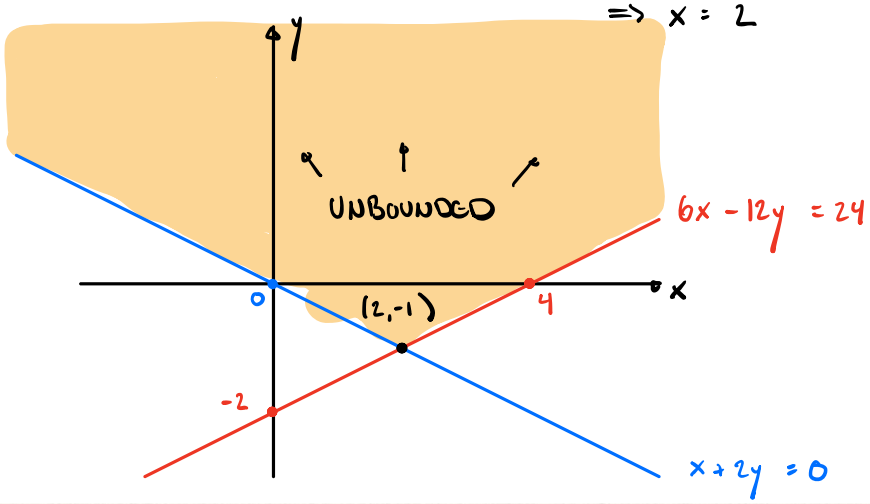
$$6x - 12y \leq 24$$
$$x + 2y \geq 0$$

BOUNDARY LINES: $6x - 12y = 24$ and $x + 2y = 0$

x-INT: $6x = 24 \Rightarrow x = 4$; $x = 0 \Rightarrow x = 0$

y-INT: $-12y = 24 \Rightarrow y = -2$; $2y = 0 \Rightarrow y = 0$

CORNER POINT: $x = -2y \Rightarrow 6(-2y) - 12y = 24 \Rightarrow y = -1 \Rightarrow x = 2$

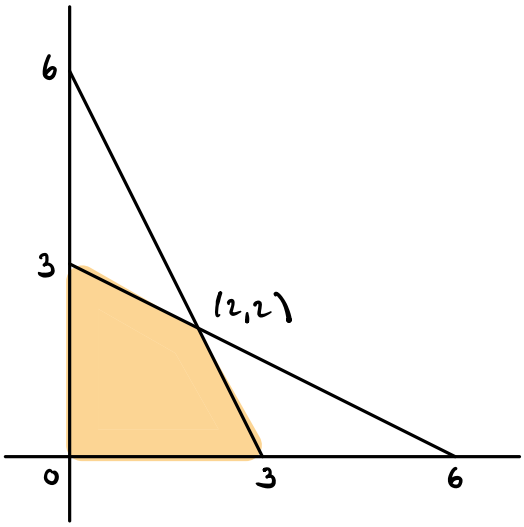


2. (15 points) Use graphical methods to maximize

$$z = 5x + 2y$$

subject to the followiing constraints.

$$6x + 3y \leq 18$$
$$x + 2y \leq 6$$
$$x \geq 0$$
$$y \geq 0$$



CORNER POINT	$z = 5x + 2y$
$(0,0)$	0
$(0,3)$	6
$(2,2)$	14
* $(3,0)$	15 *

MAX VALUE $z = 15$ WHEN $x = 3, y = 0$

3. (15 points) Consider the following problem.

> The producer of a film must rent enough vehicles to transport at least 64 crew members and at least 110 cases of equipment. Vans can be rented for 159 each and can transport 8 people and 5 cases. Trucks can be rented for 389 each and can transport 4 people and 20 cases. How many vehicles of each type should the officers rent in order to minimize the transportation costs? What are the minimal transportation costs?

Give a system of inequalities that describe the constraints on the variables in the problem above, and give an objective function to be optimized subject to those constraints. Do not do anything more. *Do not solve the problem!*

	PEOPLE	CASES	COST
VANS x	$8x$	$5x$	$159x$
TRUCKS y	$4y$	$20x$	$389y$

MINIMIZE COST $C = 159x + 389y$

SUBJECT TO CONSTRAINTS

$8x + 4y \geq 64$

$5x + 20y \geq 110$

$x \geq 0$

$y \geq 0$

4. (6 points) Suppose you need some extra money as soon as possible for an unexpeted expense. In order to help you out, your employer agrees to add \$250 to you paycheck this week, but they will then subtract \$270 from your next paycheck 2 weeks later. What annual simple interest rate is your employer charging for this loan? (Assume 1 year is 52 weeks.)

$$I = Prt \Rightarrow 20 = (250)r\left(\frac{2}{52}\right)$$

$$\Rightarrow r = \frac{52 \cdot 20}{250 \cdot 2} = 2.08 \text{ or } 208\%$$

5. (6 points) You were issued a traffic ticket and will have to pay $125. By requesting a hearing and then requesting that the hearing be rescheduled, you know that you won't have to pay the $125 until 18 months from now. How much money would you need to invest today in an account that earns 4.92% interest compounded monthly in order to have the funds to pay the traffic ticket in 18 months?

$$A = P(1+i)^n$$

$$\Rightarrow P = A(1+i)^{-n} = 125\underbrace{\left(1+\frac{.0492}{12}\right)^{-18}}_{1.0041} \approx \$116.12$$

6. (6 points) An investment of $18,000 grows to $22,625.36 over 8 years earning annual compound interest. What is the annual interest rate?

$$A = P(1+r)^n$$

$$\Rightarrow (1+r)^n = \frac{A}{P} \Rightarrow 1+r = \left(\frac{A}{P}\right)^{1/n}$$

$$\Rightarrow r = \left(\frac{A}{P}\right)^{1/n} - 1 = \left(\frac{22,625.36}{18,000}\right)^{1/8} - 1 \approx .029 \text{ or } 2.9\%$$

7. (6 points) How long would it take for for an investment to double its value if it earns 7.32% annual interest compounded quarterly?

$$A = P(1+i)^n$$

$$\Rightarrow (1+i)^n = \frac{A}{P} \Rightarrow n = \log_{(1+i)}\left(\frac{A}{P}\right)$$

$$\Rightarrow n = \frac{\ln\left(\frac{A}{P}\right)}{\ln(1+i)}$$

NOTE: INVESTMENT DOUBLES WHEN $A = 2P$, i.e. WHEN $\frac{A}{P} = 2$

$$\therefore n = \frac{\ln(2)}{\underbrace{\ln\left(1+\frac{.0732}{4}\right)}_{1.0183}} \approx 38.2 \longrightarrow$$

39 QUARTERS = 9.75 YEARS

(MUST ROUND UP BECAUSE INTEREST IS ACCRUED ONLY AT THE END OF EACH MONTH, AND 38 MONTHS IS NOT SUFFICIENT.)

8. An employer offers a 401k retirement savings account that pays 3.48% annual interest compounded monthly. By law, the maximum allowed investment per year is \$19,500. Suppose Amelia invests all \$19,500 on January 1st, and Bianca invests \$19,500/12 = \$1,625 at the end of each month for 12 months. At the end of the year ...

(a) (8 points) What is the amount to which Amelia's \$19,500 investment has grown?

(b) (8 points) What is the amount to which Bianca's \$19,500 investment has grown?

(c) (6 points) How much interest has each earned?

(a) $A = P(1+i)^n = (19{,}500)\underbrace{\left(1+\frac{.0348}{12}\right)}_{1.0029}^{12} \approx \$20{,}189.53$

(b) $FV = PMT \cdot \frac{(1+i)^n - 1}{i} = \frac{(1{,}625)(1.0029^{12} - 1)}{.0029} \approx \$19{,}814.05$

(c) A: $\$20{,}189.53 - \$19{,}500 = \$689.53$

B: $\$19{,}814.05 - \$19{,}500 = \$314.05$

9. A \$3,372.14 loan that charges 9% annual interest compounded monthly is amortized over 5 years by 60 monthly payments of \$70 (this is true).

(a) (8 points) After 2 years of payments, how much money is still owed on the loan?
Hint: This is the present value of the remaining monthly payments.

(b) (6 points) How much interest is charged over the entire 5 year loan?

(a) $PV = PMT \cdot \frac{1-(1+i)^{-n}}{i} = 170\,\frac{1-\left(1+\frac{.09}{12}\right)^{-36}}{\frac{.09}{12}}$

$= \frac{(170)(1-1.0075^{-36})}{.0075} \approx \$2{,}201.28$

(b) $\underbrace{60 \cdot 70}_{4{,}200} - 3{,}372.14 = \$827.86$