Midterm Exam 2

Fordham University
Math 1108
Math for Business: Finite

Monday 22^{nd} November, 2021

This exam contains 6 pages and 9 questions, for a total of 100 points.

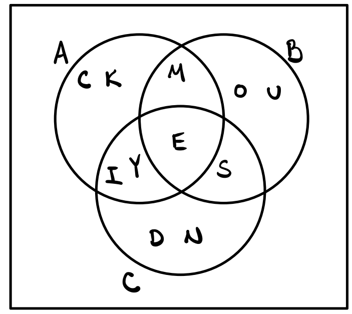
You have 75 minutes to complete this exam.

Solve all problems to the best of your ability in the distributed blue book. Begin each question on a new page, and try to keep the questions in order. Try to explain how you solved (or tried to solve) each problem, and put a box around your final answer. Your explanaition may be composed of pictures, formulas, calculations, and/or sentences.

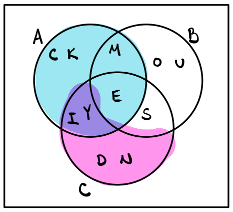
You are not required to simplify your answers. Answers can be left as expressions involving facotorials, combinations , and/or permutations.

1. Let U be the universal set and let A, B, and C be subsets of U defined as follows.

$$A = \{M, I, C, K, E, Y\}$$
$$B = \{M, O, U, S, E\}$$
$$C = \{D, I, S, N, E, Y\}$$
$$U = A \cup B \cup C$$



(a) (5 points) Find $A \cup (B' \cap C)$.



$= A \cup \{I, Y, D, N\}$

$= \boxed{\{M, I, C, K, E, Y, D, N\}}$

(b) (4 points) How many non-empty subsets of A exist?

$2^{n(A)} - 1 = \boxed{2^6 - 1 = 64 - 1 = 63}$

2. A company of 12 people must select 4 people from among themselves to work on a team.

(a) (5 points) How many ways can this be done if each team member is assigned the same role?

${}_{12}C_4 = 495$

(b) (5 points) How many ways can this be done if each team member is assigned a different role?

${}_{12}P_4 = 11,880$

(c) (5 points) How many ways can this be done if two team members are given one role and the other two team members are given a second role?

$\boxed{\underbrace{{}_{12}C_2}_{\text{CHOOSE 2 PEOPLE OUT OF 12 FOR 1ST ROLE}} \times \underbrace{{}_{10}C_2}_{\text{CHOOSE 2 PEOPLE OUT OF 10 FOR 2nd ROLE}}} = 2,970$

THEN

3. Suppose a valid password must contain 6 distinct letters (chosen from 26 lowercase letters: $a, b, c, \ldots, z$) and 3 distinct digits (chosen from 10 digits: $0, 1, 2, \ldots, 9$).

   (a) (5 points) How many valid passwords are there?

$$\boxed{\underbrace{{}_{26}C_6}_{\text{CHOOSE 6 LETTERS OUT OF 26}} \times \underbrace{{}_{10}C_3}_{\text{CHOOSE 3 DIGITS OUT OF 10}} \times \underbrace{9!}_{\text{ARRANGE THE 9 CHOSEN CHARACTERS}}}$$

THEN THEN

   (b) (5 points) How many valid passwords begin with a and end with 4?

$$\boxed{\underbrace{{}_{25}C_5}_{\text{CHOOSE 5 LETTERS OUT OF 25}} \times \underbrace{{}_{9}C_2}_{\text{CHOOSE 2 DIGITS OUT OF 9}} \times \underbrace{7!}_{\text{ARRANGE THE 7 CHOSEN CHARACTERS}}}$$

THEN THEN

4. Four socks are randomly selected from a drawer that contains 8 identical white socks, 10 identical grey socks, 12 identical black socks, and 14 identical navy socks.

   (a) (6 points) Find the probability that the socks are all different colors.

$$\frac{\text{\# WAYS TO CHOOSE ALL 4 SOCKS DIFFERENT COLORS}}{\text{\# WAYS TO CHOOSE 4 SOCKS}}$$

$$= \boxed{\frac{{}_{8}C_1 \cdot {}_{10}C_1 \cdot {}_{12}C_1 \cdot {}_{14}C_1}{{}_{44}C_4}} = \frac{8 \cdot 10 \cdot 12 \cdot 14}{{}_{44}C_4} \approx .0990$$

   (b) (6 points) Find the probability that the socks are all the same color.

$$\frac{\text{\# WAYS TO CHOOSE 4 WHITE OR 4 GREY OR 4 BLACK OR 4 NAVY SOCKS}}{\text{\# WAYS TO CHOOSE 4 SOCKS}}$$

$$= \boxed{\frac{{}_{8}C_4 + {}_{10}C_4 + {}_{12}C_4 + {}_{14}C_4}{{}_{44}C_4}} \approx .0131$$

5. Suppose $P(A) = .6$, $P(A \cap B) = .4$ and $P(A \cup B) = .7$.

   (a) (4 points) Find $P(B)$.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$.7 = .6 + P(B) - .4 \implies P(B) = .7 + .4 - .6 = .5$$

   (b) (4 points) Find $P(B|A)$.

$$P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{.4}{.6} = \frac{2}{3}$$

6. An experiment is composed of rolling two fair dice. Define the events A and B as follows.

$A =$ at least one 3 is rolled

$B =$ the sum of the dice is a multiple of 3

   (a) (5 points) Find $P(A|B)$.

S	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{3/36}{12/36} = \frac{3}{12}$$

$$= \frac{1}{4}$$

   (b) (5 points) Find $P(B|A)$.

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{3/36}{11/36}$$

$$= \frac{3}{11}$$

   (c) (5 points) Are A and B independent events? Why or why not?

$$P(A \cap B) = P(A)P(B) \ ?$$

$$\frac{3}{36} = \left(\frac{11}{36}\right)\left(\frac{12}{36}\right) \ ?$$

$$\frac{1}{12} = \left(\frac{11}{36}\right)\left(\frac{1}{3}\right) \ ?$$

$$\frac{1}{12} = \frac{11}{108} \ ?$$

$$108 = 132 \ ? \quad \text{No}$$

Alt:

$$P(A) = P(A|B) \ ?$$

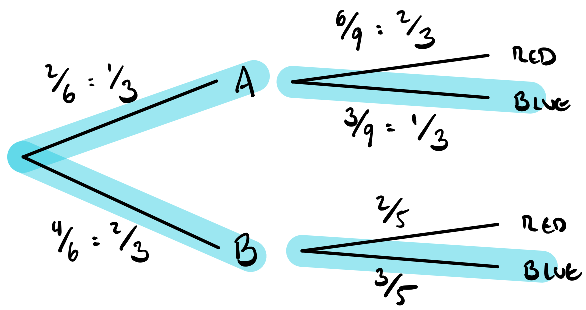
$$\frac{3}{36} = \frac{1}{4} \ ? \quad \text{No}$$

Alt:

$$P(B) = P(B|A) \ ?$$

$$\frac{12}{36} = \frac{3}{11} \ ? \quad \text{No}$$

7. (6 points) Jar A contains 6 red marbles and 3 blue marbles. Jar B contains 2 red marbles and 3 blue marbles. A single fair dice is rolled and if 1 or 2 comes up, a marble is drawn from jar A. Otherwise, it is drawn from jar B. If the drawn marble is blue, what is the probability that it comes from jar A?



$$P(A|\text{BLUE}) = \frac{P(A)P(\text{BLUE}|A)}{P(A)P(\text{BLUE}|A) + P(B)P(\text{BLUE}|B)}$$

$$= \boxed{\frac{(1/3)(1/3)}{(1/3)(1/3) + (2/3)(3/5)}} = \frac{5}{23}$$

8. Survey data collected from Amazon shoppers by a marketing agency is summerized below.

- 50% of shoppers live in an urban area, and 60% of these shoppers subscribe to Amazon Prime.
- 30% of shoppers live in a suburban area, and 80% of these shoppers subscribe to Amazon Prime.
- 20% of shoppers live in a rural area, and 30% of these shoppers subscribe to Amazon Prime.

(a) (5 points) What is the probability that a randomly selected Amazon shopper subscribes to Amazon Prime?

$$P(\text{PRIME}) = P(\text{URBAN} \cap \text{PRIME}) + P(\text{SUBURBAN} \cap \text{PRIME}) + P(\text{RURAL} \cap \text{PRIME})$$

$$= P(\text{URBAN})P(\text{PRIME}|\text{URBAN}) + P(\text{SUBURBAN})P(\text{PRIME}|\text{SUBURBAN}) + P(\text{RURAL})P(\text{PRIME}|\text{RURAL})$$

$$\boxed{\begin{aligned} &= (.5)(.6) + (.3)(.8) + (.2)(.3) \\ &= .3 + .24 + .06 = .6 \end{aligned}}$$

(b) (5 points) What is the probability that a randomly selected Amazon Prime subscriber lives in a rural area?

$$P(\text{RURAL}|\text{PRIME}) = \frac{P(\text{RURAL})P(\text{PRIME}|\text{RURAL})}{P(\text{PRIME})}$$

$$= \boxed{\frac{(.2)(.3)}{.6} = \frac{.06}{.6} = .10}$$

(c) (5 points) Are the two events "an amazon shopper lives in an urban area" and "an Amazon shopper subscribes to Amazon Prime" independent events? Why or why not?

$$P(\text{URBAN} \cap \text{PRIME}) = P(\text{URBAN})P(\text{PRIME}) \ ?$$

$$P(\text{URBAN})P(\text{PRIME}|\text{URBAN}) = P(\text{URBAN})P(\text{PRIME}) \ ?$$

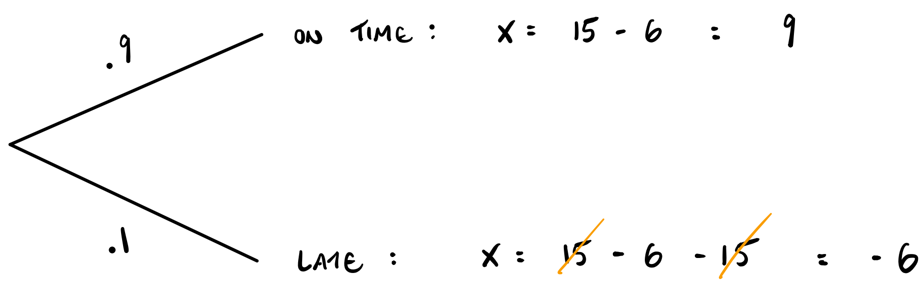
$$(.5)(.6) = (.5)(.6) \ ? \quad \text{YES}$$

$$\left(\text{NOTICE THAT } P(\text{PRIME}|\text{URBAN}) = P(\text{PRIME}) \right)$$

9. A delivery company charges a flat rate of \$15 to deliver any small package within 2 days, and they guarentee on-time delivery. That is, if a package is not delivered on-time, the company refunds the charge of \$15 to the customer. Suppose it costs the company \$6 to deliver each package (regardless of whether it is delivered on time or not), and 90% of packages are delivered on time. Define the random variable x to be the net gain/loss experienced by this company on each delivery.

   (a) (6 points) Describe the probability distribution for x by filling in a table liek the one below.

x	9	-6
$p(x)$	.9	.1



   (b) (4 points) Find the expected value $E(x)$.

$$E(x) = x_1 p(x_1) + x_2 p(x_2)$$

$$= (9)(.9) + (-6)(.1)$$

$$= 8.10 - .6 = \$7.50$$