Fordham Math 1108 **Review for Final Exam**
Math for Business: Finite **John Adamski, PhD**

Overview

The final exam is cumulative and will cover all modules/topics discussed this semester. Below is a brief overview of the topics that may appear on the final exam.

Systems of Equations/Inequalities (sections 5.1-2)

Graphing linear equations and linear inequalities, finding intersection(s) of two or more lines, graphing solution regions of systems of inequalities, generating systems of inequalities corresponding to a word problem. Note that the inequalities $x \geq 0$ and $y \geq 0$ are often necessary but not stated explicitly.

Optimization of Linear Objective Functions subject to linear constraints (section 5.3)

Optimize (maximize/minimize) a linear objective function $z = Ax + By$ subject to given constraints. Both the system of inequalities and the objective function may be described in the form of a word problem. Sketching level curves $z = c$ for arbitrary constants ($c = Ax + By$, i.e. $y = -Ax/B + c/B$). A theorem on the existence of optimal solutions states that if the solution region determined by the constraint inequalities is bounded then the maximum and minimum values of the objective function over the solution region exist and are attained at corner points of the solution region.

Simple Interest and Compound Interest (sections 3.1-2)

Accounts that earn simple interest grow linearly and accounts that earn compound interest grow exponentially. Calculate interest and total balance for pricipal investments earning simple/compound interest. Understand the difference between nominal interest rate (i.e. annual interest rate) and effective interest rate (i.e. annual percentage yield, APY). Be able to manipulate all financial math equations in order to solve for any variable, which may require the use of logarithms.

Future/Present Values of Annuities and Ammortization (sections 3.3-4)

An annuity always involves one large exchange of money in one direction and a sequence of equal-sized repeated exchanges in the other. If the large exchange of money occurs first (e.g. a bank loan) then that is the present value of the annuity and the sequence of equal-sized repeated exchanges (e.g. loan payments) will sum to more that the present value due to the interest that accumulates on the account balance. If the equal-sized exchanges of money occur first (e.g. deposits into a retirement account/sinking fund) then the large exchange of money occurs at the end of the annuity (e.g. withdrawing your savings) and that amount is the future value of the annuity, and it is larger than the sum of the equal-sized repeated payments because of the interest that accumulates on the account balance.
Note: The following formulas will be provided on the Final Exam.

Financial Math Formulas

$$I = Prt, \qquad A = (1+i)^n P$$

$$FV = PMT \cdot \frac{(1+i)^n - 1}{i}, \qquad PV = PMT \cdot \frac{1-(1+i)^{-n}}{i}$$

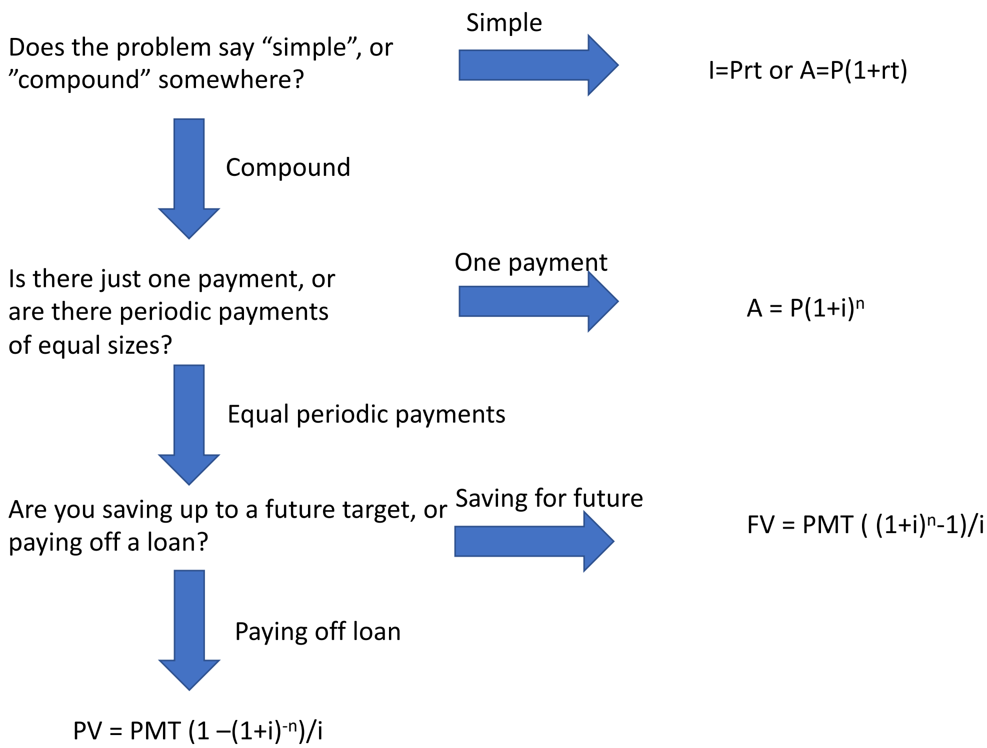

Figure 1: This flow chart summarizes how to determine which formula to use in which situation.

Counting Principals (sections 7.3-4)

Multiplicaion rule, perumutations, combinations; knowing when to use which one; knowing how to combine any/all of these principals to solve problems.

$$_nP_r = \frac{n!}{(n-r)!}, \qquad _nC_r = \frac{n!}{r!(n-r)!}$$

Introduction to Sets and Probability (sections 7.2 and 8.1-2)

Sets, subsets $A \subseteq B$ (including the empty set $\emptyset$), universal sets U, set operations (unions $A \cup B$, intersections $A \cap B$, complements A'), Venn diagrams, addition principal, sample space S (a universal set), visualizing the sample space (tables, trees, etc.), calculating probabilities of events by counting.

$$n(A \cup B) = n(A) + n(B) - n(A \cap B), \qquad P(A) = \frac{n(A)}{n(S)}$$

Conitional Probability and Bayes' Formula (sections 8.3-4)

The definition of conditional probability, the general multiplication rule and using it to calculate probabilities, independent events, mutually exclusive events, Bayes' formula.

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \qquad P(A \cap B) = P(A)P(B|A) = P(B)P(A|B), \qquad P(A|B) = \frac{P(A)P(B|A)}{P(B)}$$

Descriptive Statistics (sections 10.1-3)

Population/sample mean (μ, $\overline{x}$), median, mode, population/sample variance (σ^2, s^2), population/sample standard deviation (σ, s).

$$\mu = \overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i, \qquad s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \overline{x})^2}$$

Binomial Distributions and Normal Distributions (sections 10.4-5)

Bernoulli trials and binomial experiments, calculating probabilities for binomial experiments, calculating mean and standard deviation for binomial experiments, sketching normal distriutions with a given mean and standard deviation, the 68-95-99.7 rule

$$P(x = k) =_n C_k p^k q^{n-k}, \qquad \mu = np, \qquad \sigma = \sqrt{npq}$$

Calculating Probabilities with Normal Distributions (section 10.5)

Standardization of normal distributions (i.e. z-scores), using the table of areas under the standard normal distribution to calculate probabilities, approximating binomial distributions with a normal distribution and correcting for continuity ($\pm .5$).

$$z = \frac{x - \mu}{\sigma}$$

Practice Problems

If you want, you may turn in your answers to this practice exam for extra credit. Your score will only count toward your course grade if it raises your average. Otherwise it will not count.

Answer all 12 questions for a total of 100 points. Write your solutions neatly on paper. Show how you solved each problem and put a box around your final answers. Put your solutions in my mailbox in JMH 407 by 4pm Tuesday 12/13 (solutions will be posted at this time). Good luck!

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1. (4 points) Sophia is planning a European vacation of no more than 10 days that she will split between Athens and Barcelonia. Every day she spends in Athens she will visit two museums and two restaurants, and she will spend \$150. Every day she spends in Barcelona she will visit one museum and three restaurants, and she will spend \$210. Suppose Sophia wants to visit at least 12 museums, visit at least 18 restaurants, and spend as little money as possible on her European vacation. Let x be the number days she spends in Athens and let y be the number of days she spends in Barcelona. Assume that the time it takes to travel to/from/within Europe is negligible. Give a system of inequalities that summarizes the constraints on Sophia's planned European vacation, and give the objective function that she is trying to maximize/minimize (which one?). Do not draw any graphs or solve any equations.

2. (a) (4 points) Graph the solution region determined by the system of linear inequalities. Label all x-intercepts, y-intercepts, and corner points.

$$\begin{aligned} 3x + 2y &\geq 24 \\ y - x &\geq 2 \\ 2y - x &\leq 16 \end{aligned}$$

   (b) (2 points) Is the solution region in part (a) bounded or unbounded?

   (c) (3 points) Find the maximum and/or minimum value(s) of $z = 6x - 2y$ over the solution region found in part (a).

3. Suppose \$4,500 is depositied into a new account that earns an annual interest rate of 4.44% componded monthly.

   (a) (4 points) What is the account balance after 9 years?

   (b) (3 points) How long will it take the account to reach a balance of \$10,000?

4. Suppose you purchase a new computer for \$1,199 using a credit card that charges an annual interest rate of 22.8% compunded monthly. Your plan is to pay off this debt by making

   (a) (4 points) What size monthy payment must you make in order to pay off this debt in 6 equal-sized monthly payments?

   (b) (2 points) How much interest will you have paid by the time your debt is paid off?

5. At age 30, Dr. Smith begins depositing $1,500$ at the end of every month into an account that earns 5.88% annual interest compounded monthly.

   (a) (4 points) When Dr. Smith is 50 years old, immediately after making her 240th monthly deposit, what will be Dr. Smith's account balance?

(b) (4 points) When Dr. Smith is 50 years old, immediately after making her 240th monthly deposit, she stops making deposits and starts making equal-sized repeated withdrawals at the end of each month for the next 20 years (240 withdrawals total). What size withdrawal can she plan to make each month so that her account balance is zero after her final (240th) withdrawal, when she is 70 years old?

(c) (4 points) What is the total amount of interest that Dr. Smith earns by keeping her money in this account for 40 years as described?

6. (a) (3 points) How many different ways can 12 distinct tiles be arranged in a straight line?

   (b) (3 points) How many different ways can 3 red tiles, 4 blue tiles, and 5 yellow tiles be arranges in a straight line?

   (c) (3 points) How many ways can you select 3 red tiles, 4 blue tiles, and 5 yellow tiles from a collection of 10 red tiles, 8 blue tiles, and 7 yellow tiles?

7. Let

$$A = \{J, A, Y, B, I, R, D, S\}, \qquad B = \{J, A, W, B, O, N, E, S\}, \qquad C = \{J, A, W, L, I, N, E, S\}$$

   (a) (3 points) Find the union $A \cup B$ and the intersection $A \cap B$.

   (b) (3 points) List all subsets of the intersection $A \cap C$.

   (c) (3 points) If the universal set $U = A \cup B \cup C$, find $C' \cup B$.

8. An experiment consists of flipping three fair 6-sided dice. One die has its faces labeled 1, 1, 1, 2, 2, 2; another die has its faces labeled 3, 3, 4, 4, 4, 4; and the third die has its faces labeled 5, 6, 6, 6, 6, 6. Hint: Try using a tree diagram to visualize the possible values of each die. Since each die has faces with only 2 different numbers, the tree diagram does not need to have many branches.

   (a) (2 points) What is the probability that all three dice show odd numbers?

   (b) (2 points) What is the probability that exactly two of the dice show even numbers?

   (c) (2 points) What is the probability that the sum of the three dice is odd?

   (d) (2 points) What is the probability that two of the dice show an even number given that the sum of the three dice is odd?

   (e) (2 points) Are the events "exactly two of the dice show an even number" and "the sum of the three dice is odd" independent events?

   (f) (2 points) Are the events "exactly two of the dice show an even number" and "the sum of the three dice is odd" mutually exclusive events?

   (g) (4 points) If the experiment is repeated 15 times, what is the probability that the sum of the dice is 11 exactly 9 times?

9. (4 points) A car dealership that sells both new and used cars has three salespeople – Alexander, Brianna, and Christine.

   - 28% of all cars sold are sold by Alexander, and 38% of the cars sold by Alexander are new cars.
   - 23% of all cars sold are sold by Brianna, and 86% of the cars sold by Brianna are new cars.
   - 49% of all cars sold are sold by Christine, and 17% of the cars sold by Christine are new cars.

   If a customer purchases a new car from this dealership, what is the probability that Brianna sold them the car?

10. A sample of $n = 7$ Lake Trout were collected and their weights in lbs are recorded below.

$$76.3, \quad 82.7, \quad 92.3, \quad 72.8, \quad 85.4, \quad 82.7, \quad 88.8$$

   (a) (2 points) Calculate the sample mean $\overline{x}$.

(b) (2 points) Calculate the median.

(c) (2 points) Calculate the mode.

(d) (2 points) Calculate the sample standard deviation s.

11. Weights of chicken eggs are normally distributed with mean $\mu = 2.53$ oz and standard deviation $\sigma = .24$ oz.

    (a) (4 points) What is the probability that a randomly selected chicken egg weighs either less than 2 oz or more than 3 oz?

    (b) (4 points) What is the probability that a randomly selected chicken egg weighs between 2 and 3 oz?

12. A printer is broken in such a way that every time a page is printed, there is a 10% chance that that printed page will contain a defect.

    (a) (4 points) If 20 pages are printed, what is the probability that more than 1 page contains a defect?

    (b) (4 points) If 300 pages are printed, use a normal distribution to approximate the probability that more than 15 pages contain a defect.