Exercise §4.5

u-sub

10. $\int \sin t\sqrt{1+\cos t}\,dt$

let $u = 1+\cos t$ $\left(\frac{du}{dt} = -\sin t\right.$

$du = -\sin t\,dt$

$-du = \sin t\,dt$

$= -\int \sqrt{u}\,du$

$= -\frac{2}{3}u^{3/2} + C$

$= -\frac{2}{3}(1+\cos t)^{3/2} + C$

16. $\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$

let $u = \sqrt{x}$, $du = \frac{1}{2\sqrt{x}}\,dx$

$2du = \frac{1}{\sqrt{x}}\,dx$

$= 2\int \sin u\,du$

$= -2\cos u + C$

$= -2\cos(\sqrt{x}) + C$

17. $\int \sec^2\theta\,\tan^3\theta\,d\theta$

let $u = \tan\theta$

$du = \sec^2\theta\,d\theta$

$= \int u^3\,du$

$= \frac{1}{4}u^4 + C$

$= \frac{1}{4}\tan^4\theta + C$

$3x^2+3=3(x^2+1)$

19. $\int (x^2+1)(x^3+3x)^4\,dx$

let $u = x^3+3x$

$du = (3x^2+3)\,dx$

$= 3(x^2+1)\,dx$

$\frac{1}{3}du = (x^2+1)\,dx$

$= \frac{1}{3}\int u^4\,du$

$= \frac{1}{15}u^5 + C$

$= \frac{1}{15}(x^3+3x)^5 + C$

38. $\int_0^{\sqrt{\pi}} x\cos(x^2)\,dx$

let $u = x^2$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

If $x=\sqrt{\pi}$, $u=\pi$.

$x = 0$, $u=0$.

$= \frac{1}{2}\int_0^{\pi} \cos(u)\,du$

$= \frac{1}{2}\sin(u)\Big|_0^{\pi} = \frac{1}{2}\sin(\pi) - \frac{1}{2}\sin 0$

$= 0$

51. $\int_0^1 \frac{dx}{(1+\sqrt{x})^4}$

let $u = 1+\sqrt{x} \Rightarrow \sqrt{x} = u-1$

$du = \frac{1}{2\sqrt{x}}\,dx$

$2\sqrt{x}\,du = dx$

$\Rightarrow 2(u-1)\,du = dx$

if $x=1$, $u=1+1=2$

$x=0$, $u=1+0=1$

$= \int_1^2 \frac{2(u-1)\,du}{u^4}$

$= \int_1^2 \frac{2}{u^3} - \frac{2}{u^4}\,du$

$= -\frac{1}{u^2}\Big|_1^2 + \frac{2}{3}\frac{1}{u^3}\Big|_1^2$

$= \left(-\frac{1}{4}+1\right) + \frac{2}{3}\left(\frac{1}{8}-1\right)$

$= \frac{3}{4} - \frac{7}{12} = \frac{2}{12} = \frac{1}{6}$