

## Homework 4

Sections 3.1-3.3

Due Monday 10/24

1. Find the logistic model

$$P_{n+1} = rP_n \left(1 - \frac{P_n}{C}\right)$$

that satisfies the following conditions.

(a)  $P_0 = 5000, P_1 = 8000, P_2 = 6000$ .

(b) If  $P_0 = 5000$  then  $P_1$  is twice  $P_0$ , and if  $P_0 = 8000$  then  $P_1$  is half  $P_0$ .

2. Find Ricker model

$$P_{n+1} = rP_n e^{-P_n/N}$$

that satisfies the following conditions.

(a)  $N = 1000, P_0 = 100, P_1 = 250$ .

(b)  $P_0 = 2000, P_1 = 6000, P_2 = 4000$ .

3. The formula for the exact solution of  $x_{n+1} = (x_n - c)^a + c$ , where  $c$  is a constant, can be derived as follows"

(a) Use the substitution  $u_n = x_n - c$  to convert  $x_{n+1} = (x_n - c)^a + c$  into  $u_{n+1} = u_n^a$ .

(b) Find the exact solution for  $u_n$ .

(c) Use the substitution  $u_n = x_n - c$  again to find an exact solution for  $x_n$ .

4. For each of the following equations, find all fixed points and use a substitution of the type in question 3 to find an exact solution. Then use the exact solution to classify each fixed point as either stable or unstable, and find the basin of attraction for all stable fixed points.

(a)  $x_{n+1} = 5 + \sqrt{x_n - 5}$

(b)  $x_{n+1} = x_n^2 - 2x_n + 2$

5. For each of the following functions, find all fixed points (if any) algebraically, and use the derivative to classify each as either stable or unstable.

(a)  $f(x) = 4x^5$

(b)  $f(x) = 4xe^{-x}$

(c)  $f(x) = \frac{2}{x+1}$

(d)  $f(x) = x^{1/4}, x_0 > 0$

6. Consider the function

$$f(x) = \frac{3x - 1}{2x^2 - x + 1}$$

- Use the graph below to identify all fixed points of  $f$  and use algebra to confirm your answer.
- Create some cobweb graphs to determine the stability of each fixed point and use  $f'$  to confirm your answer.
- For each stable fixed point (if any), use cobweb graphs to determine its basin of attraction, i.e., the largest interval containing the stable fixed point  $p$  in which solutions converge to  $p$ .

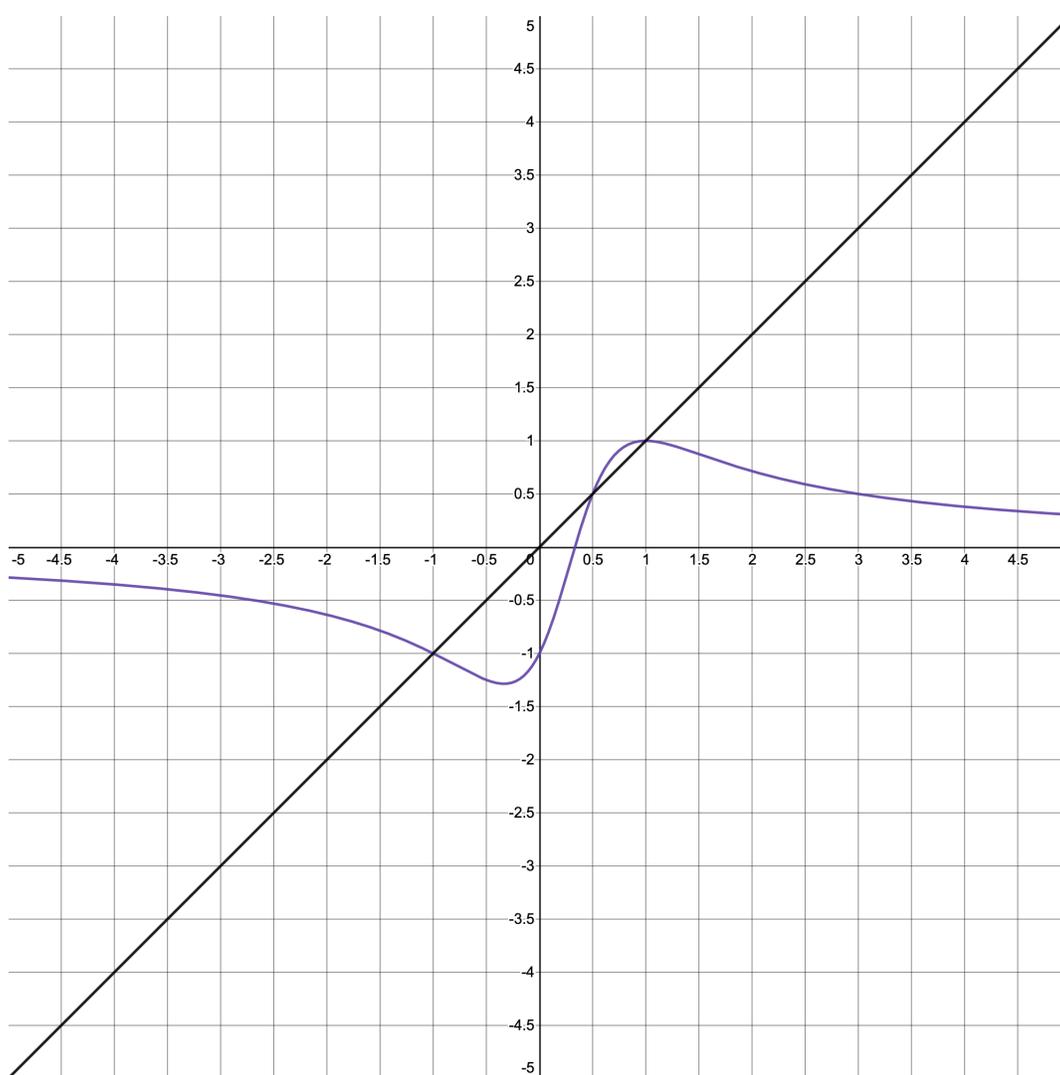


Figure 1: <https://www.desmos.com/calculator/hpaczexfzq>