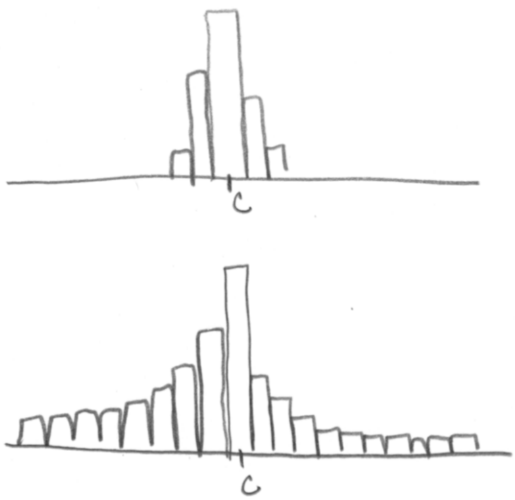
§ 2.3 MEASURES OF VARIABILITY



SAME CENTER

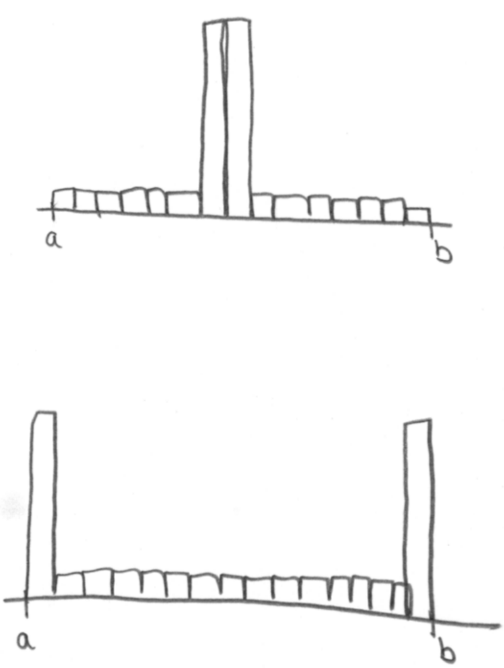
DIFFERENT DEGREES OF SPREADING OUT - VARIABILITY

(MANUFACTURERS WANT LOW VARIABILITY FOR QUALITY CONTROL)

3 WAYS TO MEASURE

1. RANGE

<u>Def</u>: THE <u>RANGE</u> R OF A SET OF n MEASUREMENTS IS DEFINED AS THE DIFFERENCE BETWEEN THE LARGEST & SMALLEST MEASUREMENTS.



SAME RANGE $b - a$

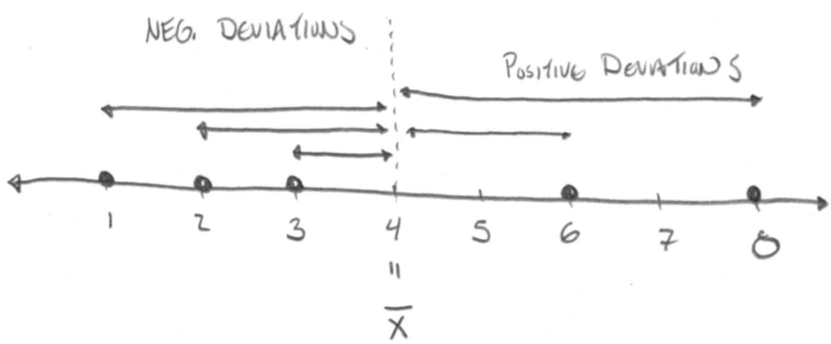
DEF: GIVEN n MEASUREMENTS, $x_1, x_2, \ldots, x_n$

WITH MEAN $\bar{x} = \sum x_i$,

THE DEVIATION OF A MEASUREMENT FROM THE MEAN

IS $x_i - \bar{x}$

e.g.



BUT WE CAN'T AVERAGE THE DEVIATIONS SINCE WE ALWAYS GET 0.

$$\left(\sum_{i=1}^{n} (x_i - \bar{x}) = \sum_{i=1}^{n} x_i - \sum_{i=1}^{n} \bar{x} = n\left(\frac{\sum x_i}{n}\right) - n\bar{x} = 0 \right)$$

DEF: VARIANCE OF A POPULATION OF N MEASUREMENTS

$$\sigma^2 = \frac{\sum_{i=1}^{N} (x_i - \mu)^2}{N}$$

NOTE: VARIANCE ≥ 0 ALWAYS

VARIANCE OF A SAMPLE OF n MEASUREMENTS

$$s^2 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1}$$

THIS DENOM. GIVES BETTER ESTIMATE OF σ^2

e.g.

$$s^2 = \frac{(-3)^2 + (-2)^2 + (-1)^2 + 2^2 + 4^2}{4}$$

$$= 8.5$$

NOTE: THE UNIT FOR VARIANCE IS THE ORIGINAL UNIT OF MEASUREMENT <u>SQUARED</u>!

<u>Def:</u> THE <u>STANDARD DEVIATION</u> OF A SET OF MEASUREMENTS IS EQUAL TO THE POSITIVE SQ. RT. OF THE VARIANCE

SAMPLE STATISTICS	POP. PARAMETERS
n = SAMPLE SIZE	N = POP. SIZE
$\bar{x}$ = SAMPLE MEAN	μ = POP. MEAN
s^2 = SAMPLE VARIANCE	σ^2 = POP. VARIANCE
$s = \sqrt{s^2}$ = SAMPLE STND. DEV.	$\sigma = \sqrt{\sigma^2}$ = POP. STND. DEV

<u>EXAMPLE</u>

<u>COMPUTING FORMULA</u>

$$s^2 = \frac{\sum x_i^2 - \frac{(\sum x_i)^2}{n}}{n-1}$$

<u>PROOF:</u> WANT TO SHOW $\sum x_i^2 - \frac{1}{n}\left(\sum x_i\right)^2 = \sum (x_i - \bar{x})^2$

$$\sum (x_i - \bar{x})^2 = \sum (x_i^2 - 2x_i\bar{x} + \bar{x}^2)$$

$$= \sum x_i^2 - 2\bar{x} \sum x_i + n\bar{x}^2$$

$$= \sum x_i^2 - 2\bar{x}(n\bar{x}) + n\bar{x}^2$$

$$= \sum x_i^2 - 2n\bar{x}^2 + n\bar{x}^2$$

$$= \sum x_i^2 - n\bar{x}^2 = \sum x_i^2 - n\left(\frac{\sum x_i}{n} \right)^2$$

$$= \sum x_i^2 - \frac{\left(\sum x_i\right)^2}{n}$$

REMARKS: LARGER VALUES FOR s^2, s $\Rightarrow$ GREATER VARIABILITY

IF $s^2 = s = 0$ THEN ALL MEASUREMENTS ARE SAME