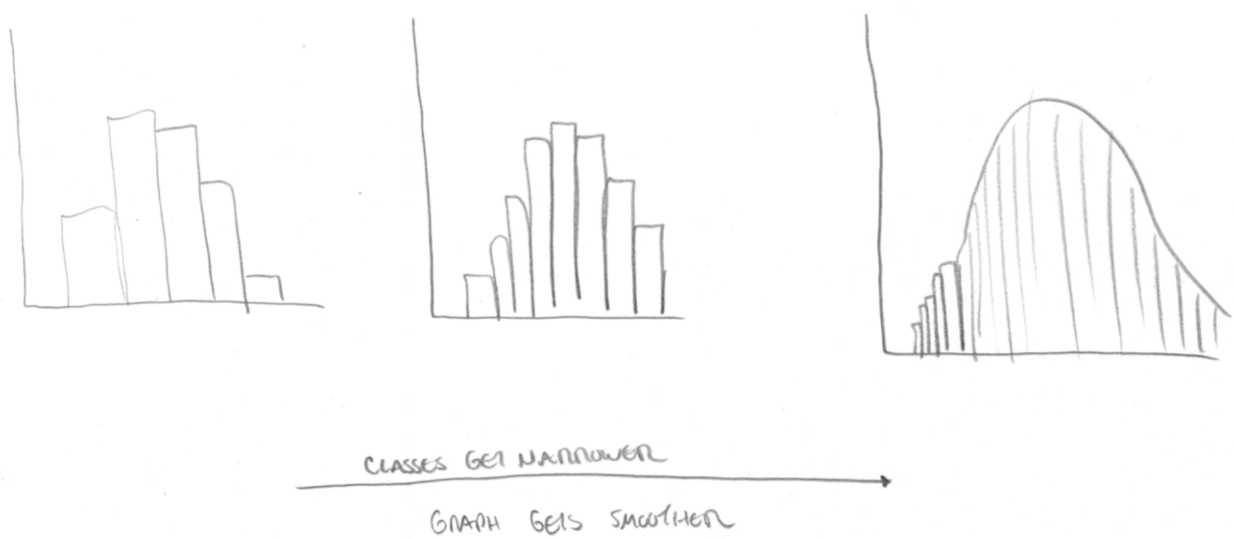
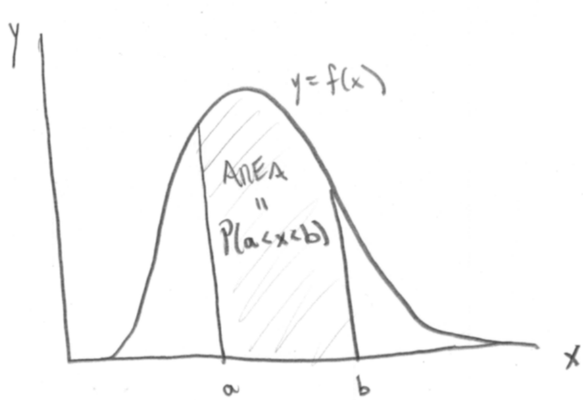
§6.1 PROBABILITY DISTRIBUTIONS FOR CONTINUOUS RANDOM VARIABLES

<u>CONTINUOUS</u> RANDOM VARIABLES TAKE ON ∞ MANY (UNCOUNTABLE!) VALUES. SO WE CAN'T ASSIGN POSITIVE PROBABILITIES $p(x)$ FOR EVERY POSSIBLE VALUE OF THE RANDOM VARIABLE, BECAUSE THE PROBABILITIES WOULD NO LONGER SUM TO 1.

<u>NEW APPROACH</u>.

MAKE HISTOGRAM FOR CONT. RAN. VAR. WITH MORE & MORE CLASSES:

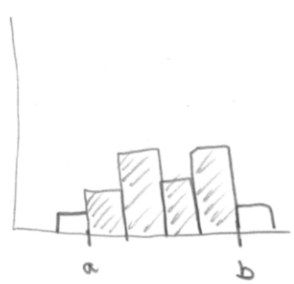


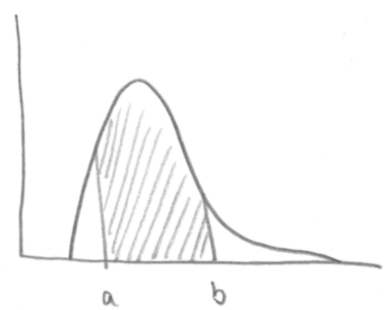


THE PROB. DISTRIBUTION IS CREATED BY DISTRIBUTING 1 UNIT OF PROBABILITY ALONG THE X-AXIS

DENSITY OF PROBABILITY VARIES WITH X. THIS MAY BE DESCRIBED BY PROB. DENSITY FUNCTION $f(x)$ (i.e. PROB. DISTRIBUTION).

DISCRETE	CONTINUOUS
1) SUM OF ALL PROB. $= 1$	1) AREA UNDER PROB. DISTRIBUTION $= 1$
2) PROBABILITY THAT X IS IN A CERTAIN INTERVAL $P(a \le x \le b)$ = SUM OF PROBABILITIES IN THAT INTERVAL	2) PROB$(a \le x \le b)$ = AREA UNDER PROB DISTRIBUTION BETWEEN a & b.





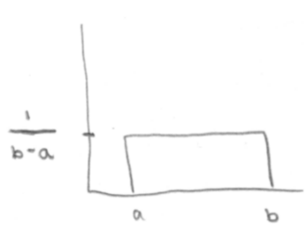
NOTE: FOR CONTINUOUS RANDOM VAR:

(i) $P(x = a) = 0$

(ii) $P(x \le a) = P(x < a)$ & $P(x \ge a) = P(x > a)$

WE MODEL PROBABILITY DISTRIBUTIONS WITH CERTAIN FUNCTIONS $f(x)$, THAT FIT THE DATA WELL.

UNIFORM



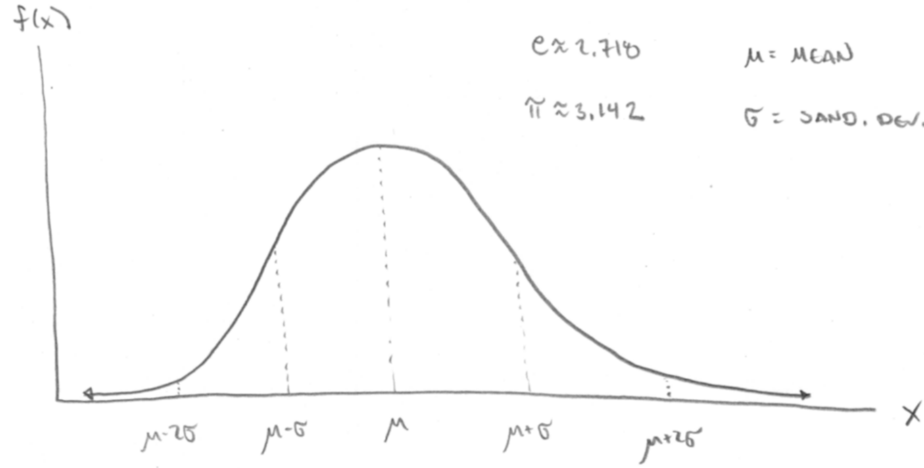
EXPONENTIAL

§6.2 THE NORMAL PROBABILITY DISTRIBUTION

NORMAL PDF : $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$, $-\infty < x < \infty$

$e \approx 2.718$ μ = MEAN

$\pi \approx 3.142$ σ = STAND. DEV.



SYMMETRIC ABOUT CENTER μ

$P(x < \mu) = P(x > \mu) = .5$

σ GETS BIGGER $\longrightarrow$ SHORTER, WIDER DISTRIBUTION

σ GETS SMALLER $\longrightarrow$ TALLER, NARROWER DISTRIBUTION.

<u>REMEMBER</u>: ALMOST ALL OBSERVATIONS LIE WITHIN 3 σ'S OF μ

i.e. SATISFY $\mu - 3\sigma < x < \mu + 3\sigma$.

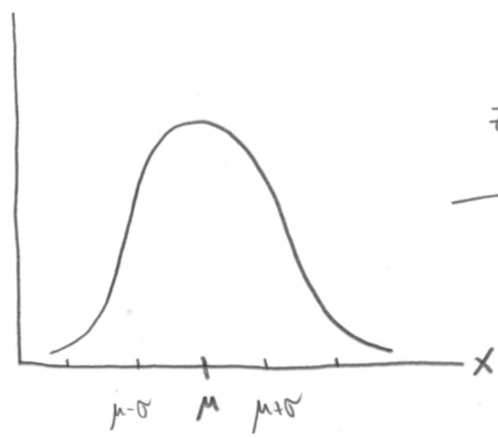
e.g. SKETCH NORMAL DISTRIBUTION WITH $\mu = 12$ AND $\sigma = 3$.

§6.3 TABULATED AREAS OF THE NORMAL PROBABILITY DISTRIBUTION

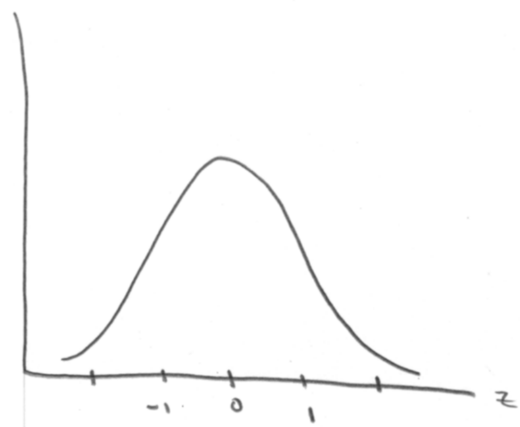
A NORMAL RANDOM VARIABLE X IS STANDARDIZED BY EXPRESSING ITS VALUE AS THE NUMBER OF STANDARD DEVIATIONS σ IT LIES TO THE LEFT (−) OR RIGHT (+) OF MEAN μ

JUST A CHANGE OF UNITS!

STANDARDIZED NORMAL RAN. VAR.

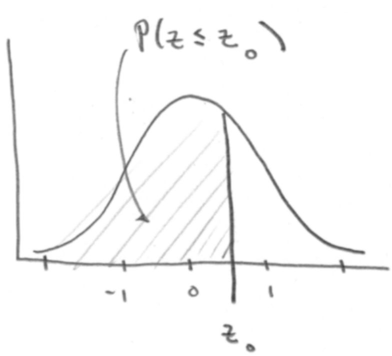


$$z = \frac{x - \mu}{\sigma}$$



$\mu = 0, \quad \sigma = 1$

CUMULATIVE AREA GIVEN IN TABLE 3 OF APPENDIX I.



e.g. $P(z \le \_\_)$

$P(z \ge \_\_)$

$P(\_\_ \le z \le \_\_)$

e.g. $P(-1 \le z \le 1)$

$P(-2 \le z \le 2)$

e.g. LET X BE NORMALLY DISTRIBUTED RAN. VAR.

WITH $\mu = 12$, $\sigma = 3$.

FIND $P(11 \leq x \leq 14)$.

FUEL ECONOMY

e.g. STUDIES SHOW GASOLINE EFFICIENCY FOR COMPACT CARS IS NORMALLY DISTRIBUTED WITH $\mu = 35.5$ MPG, $\sigma = 4.5$ M.P.G.

WHAT PERCENTAGE OF COMPACT CARS GET 40 MPG OR MORE?

e.g. IF A COMPACT CAR CLAIMS TO HAVE BETTER FUEL ECONOMY THAN 95% OF COMPETITION, WHAT MPG DOES IT GET?