§7.6 THE SAMPLING DISTRIBUTION FOR THE SAMPLE PROPORTION

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35. (a) MEAN OF SAMPLING DISTRIBUTION OF SAMPLE PROPORTION $\hat{p}$

$= p = \boxed{.3}$

S.E. $\sqrt{\frac{pq}{n}} = \sqrt{\frac{(.3)(.7)}{100}} \approx \boxed{.0458}$

(b) MEAN $= p = \boxed{.1}$

S.E. $= \sqrt{\frac{pq}{n}} = \sqrt{\frac{(.1)(.9)}{400}} = \boxed{.015}$

(c) MEAN $= p = \boxed{.6}$

S.E. $= \sqrt{\frac{pq}{n}} = \sqrt{\frac{(.6)(.4)}{250}} \approx \boxed{.0310}$

36. (a) $np = (50)(.05) = 2.5 < 5$ ✗

$\boxed{\text{No.}}$

(b) $np = (75)(.1) = 7.5 \geq 5$ ✓

$nq = (75)(.9) = 67.5 \geq 5$ ✓

$\boxed{\text{YES}}$

(c) $np = (250)(.99) = 247.5 \geq 5$ ✓

$nq = (250)(.01) = 2.5 < 5$ ✗

$\boxed{\text{No}}$

37. (a) $P(\hat{p} \le .43)$

$$z = \frac{.43 - p}{S.E.} \approx \frac{.43 - .4}{.0566} \approx .5300$$

$$S.E. = \sqrt{\frac{pq}{n}} = \sqrt{\frac{(.4)(.6)}{75}} \approx .0566$$

$= P(z \le .53) = \boxed{.7019}$

(b) $P(.35 \le \hat{p} \le .43) = P(-.88 \le z \le .53) = \boxed{.5125}$

38. (a) $np = (500)(.1) = 50 \ge 5$ ✓

$nq = (500)(.9) = 450 \ge 5$ ✓

$\boxed{\text{Yes}}$

(b) $P(\hat{p} > .12) = P(z > 1.49) = \boxed{.0681}$

$$z = \frac{.12 - .1}{\sqrt{\frac{(.1)(.9)}{500}}} \approx 1.49$$

(c) $\boxed{.5}$ $P(\hat{p} < p) = .5 \longleftrightarrow P(z < 0) = .5$

(d) $P(.08 \le \hat{p} \le .12) = P(-1.49 \le z \le 1.49) = \boxed{.8638}$

39. $SE = \sqrt{\frac{pq}{n}}$

(a) .0100 (b) .03 (c) .0458 (d) .05

(e) .0458 (f) .03 (g) .0100

(DO YOU SEE WHY THESE ARE THE SAME?)

40. (a) SINCE $np = 35$ AND $nq = 15$ (BOTH > 5)

SAMPLING DISTRIBUTION OF $\hat{p}$ IS APPROXIMATELY NORMAL

BY CENTRAL LIMIT THEOREM.

(b) MEAN $= p = \boxed{.7}$

$S.E. = \sqrt{\frac{pq}{n}} = \sqrt{\frac{(.7)(.3)}{50}} \approx \boxed{.0648}$

(c) $P(\hat{p} < .8) = P\left(z < \frac{.8-.7}{.0648}\right) \approx P(z < 1.54) = \boxed{.9382}$

41. (a) NORMAL

(b) MEAN $= p = .25$

$S.E. = \sqrt{\frac{pq}{n}} \approx \boxed{.0484}$

(c) $P(.18 \le \hat{p} \le .44) \approx P(-1.45 \le z \le 3.93) = \boxed{.9264}$

42. (a) $np = 320 > 5$ ✓

$nq = 80 > 5$ ✓

$\boxed{\text{Yes.}}$

(b) $\mu = p = .8$

$$S.E. = \sqrt{\frac{pq}{n}} = .02$$

$$P(\hat{p} > .83) = P(z > 1.5) \approx \boxed{.0668}$$

(c) $P(.76 \le \hat{p} \le .84) = P(-2 \le z \le 2) = \boxed{.9545}$

43. (a) YES (np AND nq BOTH > 5)

MEAN $\mu = p = \boxed{.78}$

$$S.E. = \sqrt{\frac{(.78)(.22)}{100}} \approx \boxed{.0414}$$

(b) $P(\hat{p} < .75) = P(z < -.72) = \boxed{.2358}$

(c) $P(.7 < \hat{p} < .75) = P(-1.93 < z < -.72) = \boxed{.2090}$

(d) SINCE $P(\hat{p} < .65) = P(z < -3.14) = .0008$,

THIS SO UNUSUAL WE MIGHT CONCLUDE THAT p IS ACTUALLY SMALLER THAN .78.

44. (a) BINOMIAL ~~RANDOM VARIABLES~~ WITH HORIZONTAL AXIS

~~#~~ SCALED BY $\frac{1}{1000}$.

CAN BE APPROXIMATED BY NORMAL DISTRIBUTION.

(b) $\mu = p = \boxed{.56}$

$S.E. = \sqrt{\frac{pq}{n}} = \sqrt{\frac{(.56)(.44)}{1000}} \approx \boxed{.0157}$

$P(\hat{p} > .6) = P(z > 2.55) = \boxed{.0054}$

(c) $P(.5 < \hat{p} < .6) = P(-3.82 < z < 2.55) = \boxed{.9945}$

(d) $P(\hat{p} \geq .7) = P(z \geq 8.92) = .\underbrace{000000000000000000}_{18 \text{ ZERO'S}}2$

SO UNUSUAL THAT YES, THIS WOULD BE GOOD EVIDENCE THAT $p \neq .56$.

45. (a) $\left.\begin{array}{l} np = (55)(.13) = 7.15 \\ nq = (55)(.87) = 47.85 \end{array}\right\} > 5$ ✓ $\boxed{\text{NORMAL}}$

(b) $\mu = p = .13$, $S.E. = \sqrt{\frac{pq}{n}} \approx .0453$

$P(\hat{p} < .20) = P(z < 1.55) = \boxed{.9394}$

(c) $P(\hat{p} > .35) = P(z > 4.86) = \boxed{.0000006}$

(d) $\mu \pm \underbrace{1.96\ SE}_{\text{"MARGIN OF ERROR"}} = .13 \pm 1.96(.0453)$

$= .13 \pm .0888$

$\boxed{[.0412\ ,\ .2188]}$

46. (a) $\mu = p = .2$

$S.E = \sqrt{\frac{pq}{n}} = .04$

$P(\hat{p} \geq .25) = P(z \geq 1.25) = \boxed{.1056}$

(b) $P(\hat{p} \leq .10) = P(z \leq -2.5) = \boxed{.0062}$

(c) $P(\hat{p} \geq .35) = P(z \geq 3.75) = \boxed{.00009 \quad \text{YES, THAT'S UNUSUAL}}$

47. (a) $np > 5\ ,\ nq > 5 \Rightarrow$ $\boxed{\text{APPROX. NORMAL DISTRIBUTION}}$

MEAN $\mu = p = \boxed{.75}$, $S.E. = \sqrt{\frac{pq}{n}} \approx \boxed{.0306}$

(b) $P(\hat{p} > .8) = P(z > 1.63) = \boxed{.0516}$

(c) $p \pm \underbrace{1.96(S.E.)}_{\text{"MARGIN OF ERROR"}} = .75 \pm 1.96(.0306) = \boxed{[.69\ ,\ .81]}$