

§ 8.5 INTERVAL ESTIMATION

SUPPOSE I WANT TO KNOW THE AVERAGE WEIGHT OF A COFFEE BEAN.

I GATHER A RANDOM SAMPLE $n = 50$

AND FIND $\bar{x} = 80 \text{ mg}$, $s = 1.5 \text{ mg}$.

CLT $\Rightarrow \bar{X}$ HAS APPROX. NORMAL DISTR. WITH MEAN = POP. MEAN (μ)

USE S A ESTIMATOR (POINT) FOR σ , so S.E. = $\frac{1.5}{\sqrt{50}} \approx .2121$.

PROBABILITY \bar{X} IS WITHIN 1 S.E. OF μ IS .6826
 2 S.E. .9544

↓ EQUIVALENTLY

PROBABILITY μ IS WITHIN 1 S.E. OF \bar{X} IS .6826
 2 S.E. .9544

↓ EQUIVALENTLY

WE ARE 68.26% CONFIDENT μ IS WITHIN 1 S.E. OF \bar{X}
 WITHIN .2121 OF 80

↑
 CONFIDENCE COEFFICIENT
 (DECIMAL)
 ↓

IS BETWEEN 79.79 AND 80.21, [79.79, 80.21]

WE ARE 95.44% CONFIDENT μ IS WITHIN 2 S.E. OF \bar{X}
 WITHIN 3 S.E. OF 80

↑
 CONFIDENCE
 INTERVAL

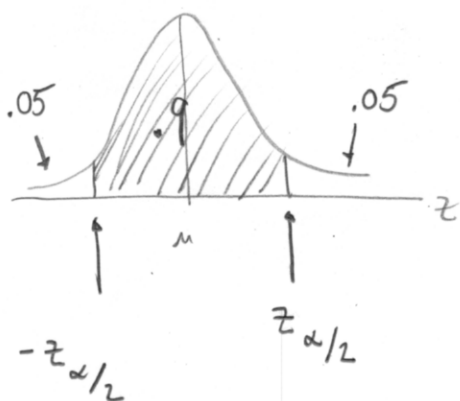
IS BETWEEN 79.58 & 80.42. [79.58, 80.42]

CONFIDENCE COEFFICIENT	(COMPLEMENT) CORRESPONDING α	CORRESPONDING $z_{\alpha/2}$
.9	.1	1.645
.95	.05	1.96
.98	.02	2.33
.99	.01	2.58

e.g. suppose $n=50$, $\bar{x}=80$, $s=1.5$.

FIND 90% CONFIDENCE INTERVAL FOR μ . ($\alpha=.1$)

99% CONFIDENCE INTERVAL FOR μ . ($\alpha=.01$)



$$\text{WE SEE } P(z \leq z_{\alpha/2}) = .95$$

$$\Rightarrow z_{\alpha/2} = 1.645$$

$$[\bar{x} - 1.645 \sigma, \bar{x} + 1.645 \sigma] \approx [\bar{x} - 1.645 s, \bar{x} + 1.645 s]$$

= [

CONFIDENCE INTERVALS FOR
POPULATION PROPORTIONS

$$\hat{p} \pm z_{\alpha/2} \sqrt{\frac{pq}{n}}$$

\uparrow
 $\frac{\bar{x}}{n}$

\downarrow
 $\sigma \times \sqrt{\frac{\hat{p}\hat{q}}{n}} = \text{S.E.}$

ex. # 34, 37