Name: * ANSWER KEY * 12/16/2019

Math 17300-FG Introduction to Probability and Statistics **Final Exam**

This exam lasts 2 hours and 15 minutes. Please silence and put away your cell phone. You are allowed 1 sheet of notes (front and back) and a calculator. Show enough work that it is clear how you arrived at your answer. Decimal answers should be rounded to 4 decimal points. Put a box around your final answer to each question. Good luck!

1. Consider the following *sample* of measurements.

80 91 53 53 46 25 92 48 7

(a) (2 points) Compute the mean $\overline{x}$, and show how you arrived at your answer.

$$\bar{x} = \frac{\Sigma x}{n} = \frac{80+91+53+53+46+25+92+48+7}{9} = \frac{495}{9} = \boxed{55}$$

(b) (2 points) Compute the median.

7 25 46 48 $\boxed{53}$ 53 80 91 92

(c) (2 points) Compute the mode.

$\boxed{53}$

(d) (2 points) Compute the range.

MAX - MIN = $92 - 7 = \boxed{85}$

(e) (2 points) Compute the variance s^2.

x	$x - \bar{x}$	$(x-\bar{x})^2$
7	-49	2304
25	-30	900
46	-9	81
48	-7	49
53	-2	4
53	-2	4
80	25	625
91	36	1296
92	37	1369

$$s^2 = \frac{\Sigma(x-\bar{x})^2}{n-1} = \frac{2304 + 900 + \ldots 1369}{9-1}$$

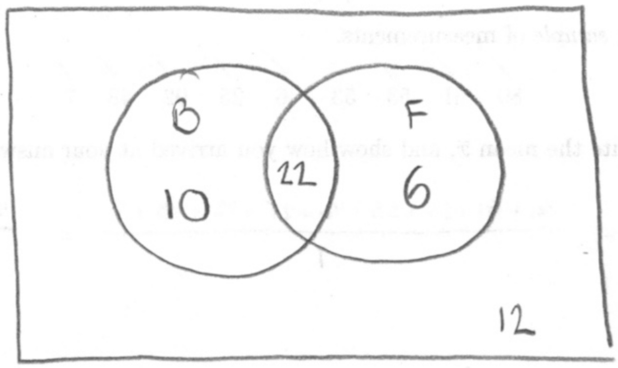
$$= \frac{6632}{8} = \boxed{829}$$

(f) (2 points) Compute the standard deviation s.

$$s = \sqrt{s^2} = \sqrt{829} \approx \boxed{28.7924}$$

2. A small school has a baseball team and football team. However, the school has only 50 students. Thus, some students play more than one sport.

- 32 students play baseball
- 28 students play football
- 12 students no not play either baseball or football



(a) (2 points) How many students play only baseball? 10

(b) (2 points) How many students play only football? 6

(c) (2 points) How many students play both baseball and football? 22

3. (a) (3 points) How many ways are there for a 12 member committee to choose a president, vice-president, and secretary?

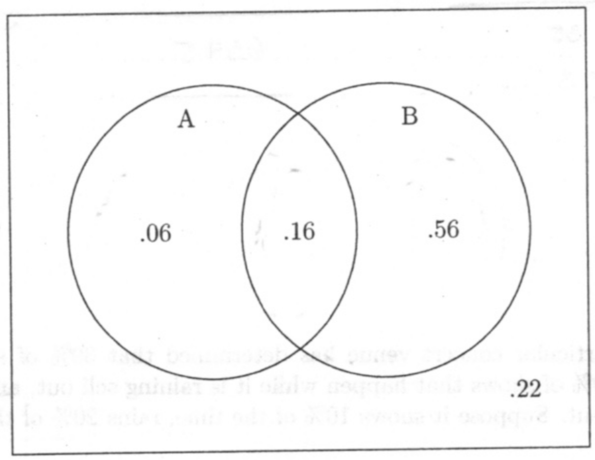
$$P^{12}_{3} = \frac{12!}{(12-3)!} = 12 \cdot 11 \cdot 10 = \boxed{1320}$$

(b) (3 points) How many ways are there for a 12 member committee to choose 5 members to serve on a subcommittee?

$$C^{12}_{5} = \frac{12!}{5!(12-5)!} = \boxed{792}$$

4. An experiment can result in events A, B, both A and B, or neither with the following probabilities. (Note: the chart and the Venn diagram are equivalent.)

	A	A'
B	.16	.56
B'	.06	.22



(a) (3 points) Find $P(A)$.

$P(A) = P(A \cap B) + P(A \cap B^c) = .16 + .06 = \boxed{.22}$

(b) (3 points) Find $P(A|B)$.

$P(A|B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{.16}{.16 + .56} = \dfrac{.16}{.72} = \boxed{.2222}$

(c) (3 points) Are A and B independent? Why or why not?

$\boxed{\text{No. } P(A) \neq P(A|B) \quad .22 \neq .2222}$

Also $P(A \cap B) \neq P(A)P(B)$

$.16 \neq (.22)(.72)$

$.16 \neq .1584$

(d) (3 points) Are A and B mutually exclusive? Why or why not?

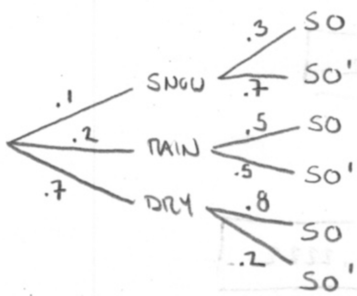
$\boxed{\text{No. } P(A \cap B) \neq 0 \quad .16 \neq 0}$

5. (4 points) Suppose you randomly select 3 animals from a group of 8 lions, 11 tigers, and 16 bears. What is the probability that you select exactly 1 lion, 1 tiger, and 1 bear?

$$\frac{C_1^8 \, C_1^{11} \, C_1^{16}}{C_3^{35}} = \frac{(8)(11)(16)}{6545} \approx \boxed{.2151}$$

6. The owner of a particular concert venue has determined that 30% of shows that happen when it is snowing sell out, 50% of shows that happen while it is raining sell out, and 80% of shows that happend while it is dry sell out. Suppose it snows 10% of the time, rains 20% of the time, and is dry 70% of the time.

(a) (4 points) Find the total probablity that a concert at this venue sells out.



$$P(SO) = P(\text{SNOW})P(SO \mid \text{SNOW}) + P(\text{RAIN})P(SO \mid \text{RAIN}) + P(\text{DRY})P(SO \mid \text{DRY})$$

$$= (.1)(.3) + (.2)(.5) + (.7)(.8)$$

$$= \boxed{.69}$$

(b) (4 points) Now suppose a particular show sold out. Find the probability that it rained that night.

$$P(\text{RAIN} \mid SO) = \frac{P(\text{RAIN})P(SO \mid \text{RAIN})}{P(SO)}$$

$$= \frac{(.2)(.5)}{.69} \approx \boxed{.1449}$$

7. (4 points) Suppose a test for certain disease is said to be 99% accurate for the following reasons.

- If a person has the disease, the probability that they will test positive is .99.
- If a person does not have the disease, the probability that they will test negative (i.e. not positive) is .99.

If the probability that a person has the disease is only .005, find the probability that a person who tests positive for the disease actually has the disease.

LET P = TEST POSITIVE

D = HAVE DISEASE

GIVEN: $P(P|D) = .99$

$P(P^c|D^c) = .99 \Rightarrow P(P|D^c) = .01$

$P(D) = .005 \Rightarrow P(D^c) = .995$

$$P(D|P) = \frac{P(D)P(P|D)}{P(D)P(P|D) + P(D^c)P(P|D^c)} = \frac{(.005)(.99)}{(.005)(.99) + (.995)(.01)}$$

$$= \boxed{.3322}$$

$\frac{.00495}{.0149}$

8. (a) (5 points) If you roll a regular die (faces: 1, 2, 3, 4, 5, 6) 20 times, what is the probability of rolling a six exactly 4 times?

BINOMIAL EXPERIMENT: $n = 20$

$p = \frac{1}{6}$

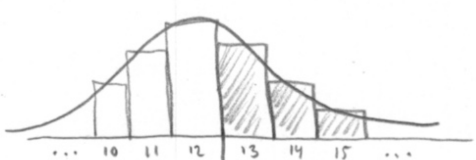
$q = \frac{5}{6}$

x = # SUCCESSES IN 20 TRIALS

$$P(x=4) = C^{20}_{4}\left(\frac{1}{6}\right)^4\left(\frac{5}{6}\right)^{16}$$

$$= \boxed{.2022}$$

(b) (5 points) If you roll a regular die 60 times, what is the probability of rolling a six more than 12 times? Use a normal distribution to approximate this binomial probability.



$$P(x > 12) \approx P(x_{NORM} \geq 12.5)$$

$$= 1 - P\left(z \leq \frac{12.5 - 10}{2.8868}\right)$$

$$= 1 - P(z \leq .87)$$

$$= 1 - .8078$$

$$= \boxed{.1922}$$

MEAN $\mu = np = (60)\left(\frac{1}{6}\right) = 10$

$\sigma = \sqrt{npq} = \sqrt{(60)\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)} \approx 2.8868$

9. A raffle is being held in which 2,000 tickets are sold for \$10 each. There is 1 top prize of \$5,000, 4 middle prizes of \$500 each, and 10 lower prizes of \$100 each. All other tickets receive no prize (\$0). Let x equal the net gain/loss from buying one ticket, that is

$$x = \text{prize money} - 10.$$

(a) (4 points) Describe the probability distribution $p(x)$ by filling in the chart below.

x	4990	490	90	-10
$p(x)$	$\frac{1}{2000}$	$\frac{4}{2000}$	$\frac{10}{2000}$	$\frac{1985}{2000}$
	.0005	.0020	.0050	.9925

(b) (4 points) Calculate the expected value $\mu = E[x]$ for x.

$$E[x] = \sum x\,p(x)$$

$$= 4990\left(\frac{1}{2000}\right) + 490\left(\frac{4}{2000}\right) + 90\left(\frac{10}{2000}\right) - 10\left(\frac{1985}{2000}\right)$$

$$= 2.495 + .98 + .45 - 9.925$$

$$= \boxed{-6}$$

10. Let z be a random variable with the standard normal probability distribution ($\mu = 0$, $\sigma = 1$). Use the table provided at the end of the exam or a calculator to answer the following questions.

(a) (2 points) Find the probability $P(z \leq -0.83)$

	.03
-0.8	.2033

$\boxed{.2033}$

(b) (2 points) Find the probability $P(z \geq 1.44)$

	.04
1.4	.9251

$1 - .9251 = \boxed{.0749}$

(c) (2 points) Determine the value z_0 such that $P(z \leq z_0) = .281$.

	.08
-0.5	.2810

$\boxed{-.58}$

(d) (2 points) Determine the value z_0 such that $P(z \geq z_0) = .011$.

$1 - .011 = .9890$

	.09
2.2	.9890

$\boxed{2.29}$

11. Suppose that the weight of chicken eggs is normally distributed with a mean $\mu = 2.03$ oz and standard deviation of $\sigma = .24$ oz.

(a) (5 points) Chicken eggs that weight between 2.15 oz and 2.35 oz are labelled "Extra Large" by the USDA. What percentage of all chicken eggs could be labelled "Extra Large"?

Let x = WEIGHT OF RANDOM CHICKEN EGG

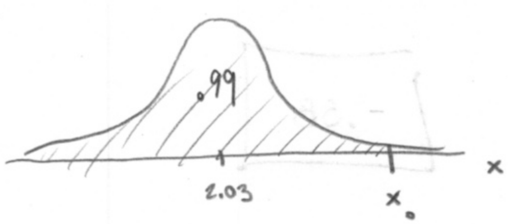
$$P(2.15 \leq x \leq 2.35) = P\left(\frac{2.15 - 2.03}{.24} \leq z \leq \frac{2.35 - 2.03}{.24} \right)$$

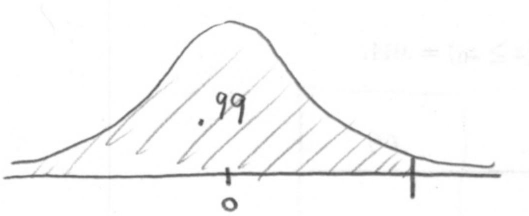
$$= P(.5 \leq z \leq 1.33)$$

$$= P(z \leq 1.33) - P(z \leq .5)$$

$$= .9082 - .6915 = \boxed{.2167}$$

(b) (5 points) How much does a chicken egg need to weigh in order to be heavier than 99% of all chicken eggs?





$z_0 = \underline{\underline{2.33}}$

$$z_0 = \frac{x_0 - \mu}{\sigma}$$

$$x_0 = \mu + z_0 \sigma$$

$$x_0 = 2.03 + 2.33(.24)$$

$$= \boxed{2.5892}$$

12. (5 points) Suppose the amount of time it takes all City College students to complete a particular exam is normally distributed with mean $\mu = 105$ minutes and standard deviation $\sigma = 12$ minutes. Find the probability that a random sample of 40 City College students take an average of $\overline{x} = 109$ minutes or more to complete the exam.

CLT $\Rightarrow$ $\overline{x}$ IS NORMALLY DISTR. WITH MEAN $\mu = 105$

AND STAND. DEV. $S.E. = \frac{\sigma}{\sqrt{n}} = \frac{12}{\sqrt{40}} \approx 1.8974$

$$P(\overline{x} \geq 109) \approx P\left(z \geq \frac{109-105}{1.8974}\right) = 1 - P(z \leq 2.11)$$

$$= 1 - .9826 = \boxed{.0174}$$

13. Does Mars, Incorporated use the same proportion of red candies in its plain and peanut varieties? A random sample of 56 plain M&M's contained 12 red candies, and another random sample of 32 peanut M&M's contained 8 red candies.

(a) (5 points) Construct a 95% confidence interval for the difference in the proportion of red candies for the plain and peanut varieties.

LET $\hat{p}_1 = \frac{x_1}{n_1} = \frac{12}{56} \approx .2143$ CLT $\Rightarrow$ $\hat{p}_1 - \hat{p}_2$ IS NORM. DISTR. WITH

$\hat{p}_2 = \frac{x_2}{n_2} = \frac{8}{32} = .25$ MEAN $p_1 - p_2$, $S.E. = \sqrt{\frac{p_1 q_1}{n_1} - \frac{p_2 q_2}{n_2}}$

$$S.E. \approx \sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}} = \sqrt{\frac{(.2143)(.7857)}{56} + \frac{(.25)(.75)}{32}} \approx .0942$$

$$p_1 - p_2 \approx \hat{p}_1 - \hat{p}_2 \pm z_{\alpha/2}\, S.E. = .2143 - .25 \pm 1.96\,(.0942)$$

$$= -.0357 \pm .1846 \quad \boxed{[-.2203, .1489]}$$

(b) (2 points) Based on the confidence interval in part (a), can you conclude that there is a difference in the proportions of red candies for the plain and peanut varieties? Explain.

$\boxed{\text{NO, THE INTERVAL CONTAINS 0.}}$

TABLE 3 *(continued)*

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998