Name: *Answer Key* 11/25/2019

Mat 173-FG Quiz 4

Please put away all papers and electronic devices except for a calculator. Show enough work that it is clear how you arrived at your answer. Put a box/circle around your final answer to each question, rounded to 4 decimal places. Good luck!

1. Let $z$ be a random variable with the standard normal probability distribution ($\mu = 0$, $\sigma = 1$). Using the table provided at the end of the exam or a calculator, determine the following probabilities.

   (a) (8 points) $P(z \leq -1.21)$

   | | .01 |
   |---|---|
   | -1.2 | .1131 |

   $\boxed{.1131}$

   (b) (8 points) $P(z \geq 0.54)$

   $= 1 - P(z \leq .54)$

   | | .04 |
   |---|---|
   | .5 | .7054 |

   $= 1 - .7054$

   $= \boxed{.2946}$

   (c) (8 points) $P(-1.21 \leq z \leq 0.54)$

   $= P(z \leq .54) - P(z \leq -1.21)$

   $= .7054 - .1131 = \boxed{.5923}$

2. Let $z$ be a random variable with the standard normal probability distribution ($\mu = 0$, $\sigma = 1$). Use the table provided at the end of the exam or a calculator to answer the following questions.

   (a) (8 points) Determine the value $z_0$ such that $P(z \leq z_0) = .025$.

   | | .06 |
   |---|---|
   | -1.9 | .0250 |

   $\boxed{-1.96}$

   (b) (8 points) Determine the value $z_0$ such that $P(z \geq z_0) = .305$.

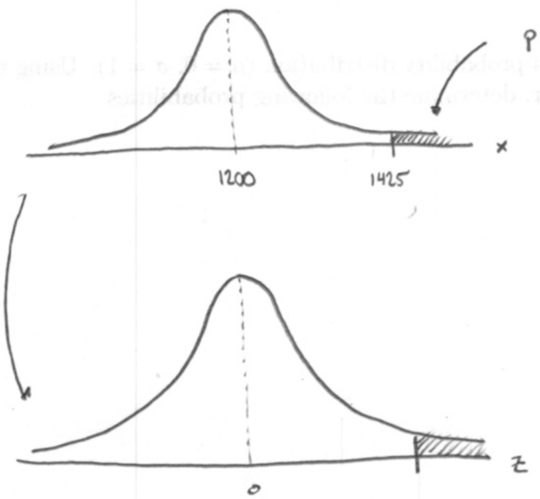
   $\Rightarrow P(z \leq z_0) = 1 - .305 = .695$

   | | .01 |
   |---|---|
   | .5 | .6950 |

   $\boxed{.51}$

3. Suppose that you must establish regulations concerning the maximum number of people who can occupy an elevator. A study indicates that if eight people occupy the elevator, the probability distribution of the total weight $x$ of the eight people is normally distributed with a mean $\mu = 1200$ pounds and a standard deviation $\sigma = 99$ pounds.

(a) (10 points) What is the probability that the total weight $x$ of eight people exceeds 1425 pounds?




$z = \frac{x - \mu}{\sigma}$

$P(x \geq 1425) = P(z \geq 2.27)$

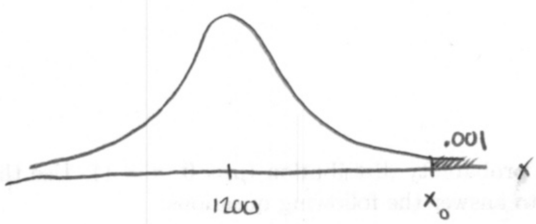
$= 1 - P(z \leq 2.27)$

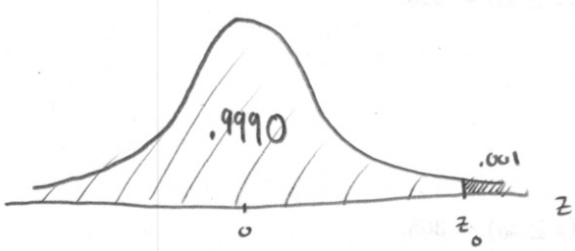
$= 1 - .9884$

$= \boxed{.0116}$

$\frac{1425 - 1200}{99} = 2.2727...$

(b) (10 points) Determine the value $x_0$ such that the probabilty that the total weight $x$ of the eight people exceeds $x_0$ is .001.




$z_0 = \frac{x_0 - \mu}{\sigma}$

$3.09 = \frac{x_0 - 1200}{99}$

$99(3.09) = x_0 - 1200$

$x_0 = 1200 + 99(3.09)$

$= \boxed{1505.91}$

| | .08 |
|---|---|
| 3.0 | .9990 |

$z_0 = 3.08, 3.09$, or $3.10$

$\left( \begin{array}{l} P(z \geq 3.09) = .001 \\ P(x \geq \underline{\ ?\ }) = .001 \end{array} \right)$

4. (12 points) Airlines and hotels often grant reservations in excess of capacity to minimize losses due to no-shows. Suppose the records of a hotel show that, on the average, $p = 10\%$ of their prospective guests will not claim their reservation (no-shows). If the hotel accepts $n = 215$ reservations and there are only 200 rooms in the hotel, what is the probability that all guests who arrive to claim a room will receive one? In other words, what is the probability that the number of no-shows $x$ is at least 15? Use a normal approximation to the binomial distribution for $x$ to answer this question.


$X \approx X_{NORM}$

$$P(x \geq 15) \approx P(X_{NORM} \geq 14.5) = P\left(z \geq \frac{14.5 - 21.5}{\sqrt{19.35}}\right)$$

$$\left(\begin{array}{l} \mu = np = (215)(.1) = 21.5 \\ \sigma = \sqrt{npq} = \sqrt{(215)(.1)(.9)} = \sqrt{19.35} \end{array}\right)$$

$$= 1 - P\left(z \leq \frac{14.5 - 21.5}{\sqrt{19.35}}\right)$$

$$\approx 1 - P(z \leq -1.59) = 1 - .0559$$

$$= \boxed{.9441}$$

5. Consider the population of all City College students and their GPA's. Assume that this population has a GPA mean $\mu = 3.34$ and standard deviation $\sigma = 0.28$. Let $\overline{x}$ be the mean GPA for a random sample of 40 City College students.

(a) (8 points) Find the mean and standard error for $\overline{x}$.

MEAN $= \mu = \boxed{3.34}$

S.E. $= \frac{\sigma}{\sqrt{n}} = \frac{.28}{\sqrt{40}} \approx \boxed{.0443}$

(b) (8 points) Find the probability that a random sample of 40 City College students has a mean GPA below 3.30.

$P(\bar{x} \le 3.30) = P\left(z \le \frac{3.30 - 3.34}{.0443}\right)$

$\approx P(z \le -.90)$

$= \boxed{.1841}$

6. (12 points) A studious bartender has observed that on average, 15% of customers do not leave a tip. On a particularly busy night, the bartender serves 250 people. Find the probability that more than 20% of her customers this night do not leave a tip.

GIVEN $p = .15$ FIND $P(\hat{p} \geq .2)$

$q = .85$

$n = 250$

MEAN FOR $\hat{p}$ IS $p = .15$

S.E. FOR $\hat{p}$ IS $\sqrt{\frac{pq}{n}} = \sqrt{\frac{(.15)(.85)}{250}} \approx .0226$

$$z = \frac{\hat{p} - p}{S.E.} = \frac{.2 - .15}{.0226} \approx 2.21$$

$$P(\hat{p} \geq .2) = P(z \geq 2.21) = 1 - P(z \leq 2.21)$$

$$= 1 - .9864$$

$$= \boxed{.0136}$$