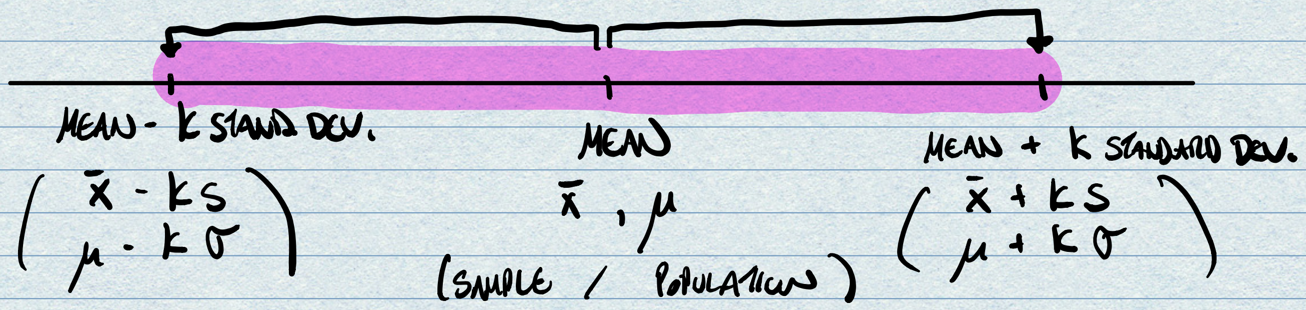
§ 2.4 PRACTICAL SIG. OF STANDARD DEVIATION

CHEVYCHEV'S THM ✓

PROPORTION OF DATA THAT LIES WITHIN K STANDARD DEVIATIONS OF THE MEAN ...



... IS AT LEAST $1 - \dfrac{1}{k^2}$.

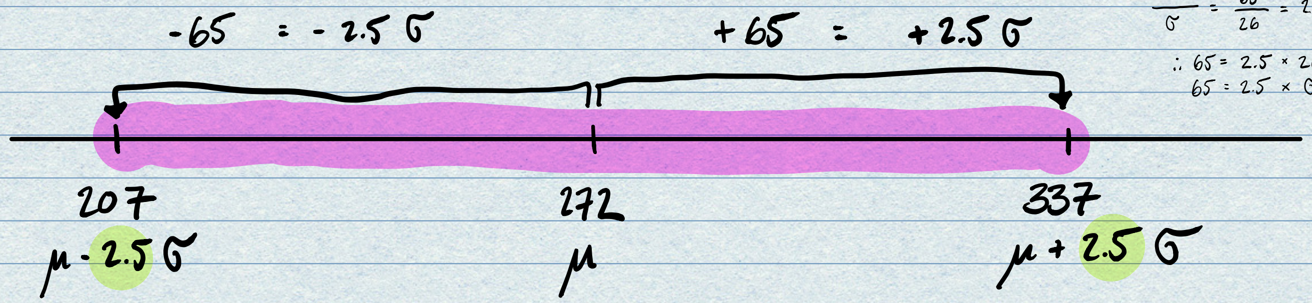
<u>ex.</u> A PLAYLIST OF 240 SONGS HAS A MEAN SONG LENGTH OF 272 SEC. THE STANDARD DEVIATION FOR THE LENGTH OF SONGS IN ~~PLAYLIST~~ IS 26 SEC.

HOW MANY SONGS HAVE A LENGTH BETWEEN 207 SEC. & 337 SEC ? (USE CHEV. THM)

$\frac{65}{\sigma} = \frac{65}{26} = 2.5$

$\therefore 65 = 2.5 \times 26$

$65 = 2.5 \times \sigma$



CHEV. THM WITH $k = 2.5$

PROPORTION OF DATA THAT LIE WITHIN 2.5 σ OF μ

IS AT LEAST

$$1 - \frac{1}{2.5^2} = .84$$

THAT IS AT LEAST 84% OF THE SONGS HAVE A LENGTH BETWEEN 207 SEC & 337 SEC.

$$84\% \text{ of } 240 = (.84)(240) = 201.6 \text{ SONGS}$$

(ROUND UP)

AT LEAST 202 SONGS HAVE LENGTH BETWEEN 207 & 337 SEC.

CHEV. THM APPLIES TO <u>ALL</u> DATA DISTRIBUTIONS.

IF DATA HAS A DISTRIBUTION THAT IS

(1) UNIMODAL

(2) SYMMETRIC

THEN WE CAN APPLY ...

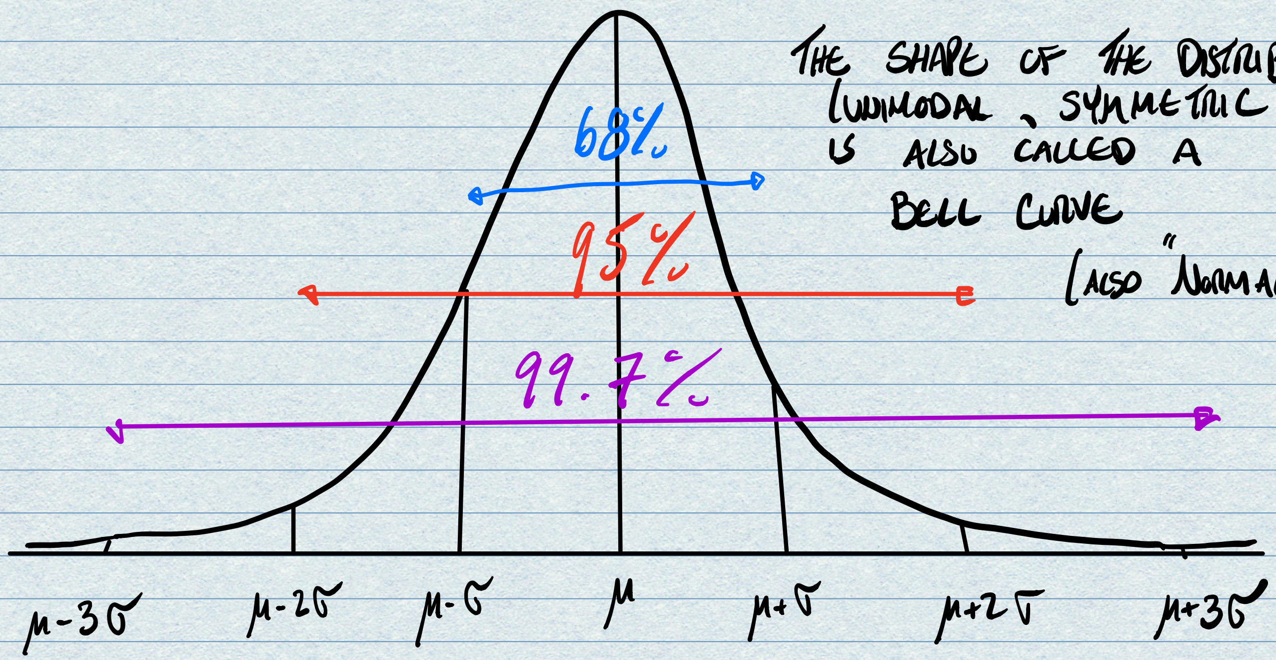
THE EMPERICAL RULE :

APPROX. 68% OF DATA LIES WITHIN 1 STAND. DEV. OF MEAN

APPROX. 95% OF DATA LIES WITHIN 2 STAND. DEV. OF MEAN

APPROX. 99.7% OF DATA LIES WITHIN 3 STAND. DEV. OF MEAN

THE SHAPE OF THE DISTRIBUTION (UNIMODAL, SYMMETRIC) IS ALSO CALLED A BELL CURVE (ALSO "NORMAL")



ex. $\mu = 272$, $\sigma = 26$

ASSUME THE LENGTH OF SONGS HAS A DISTRIBUTION THAT IS UNIMODAL & SYMMETRIC.

APPROXIMATELY HOW MANY SONGS ~~HAVE A LENGTH~~

(a) BETWEEN 220 & 324 SEC? 95% of 240 = $(.95)240 = \underline{228}$

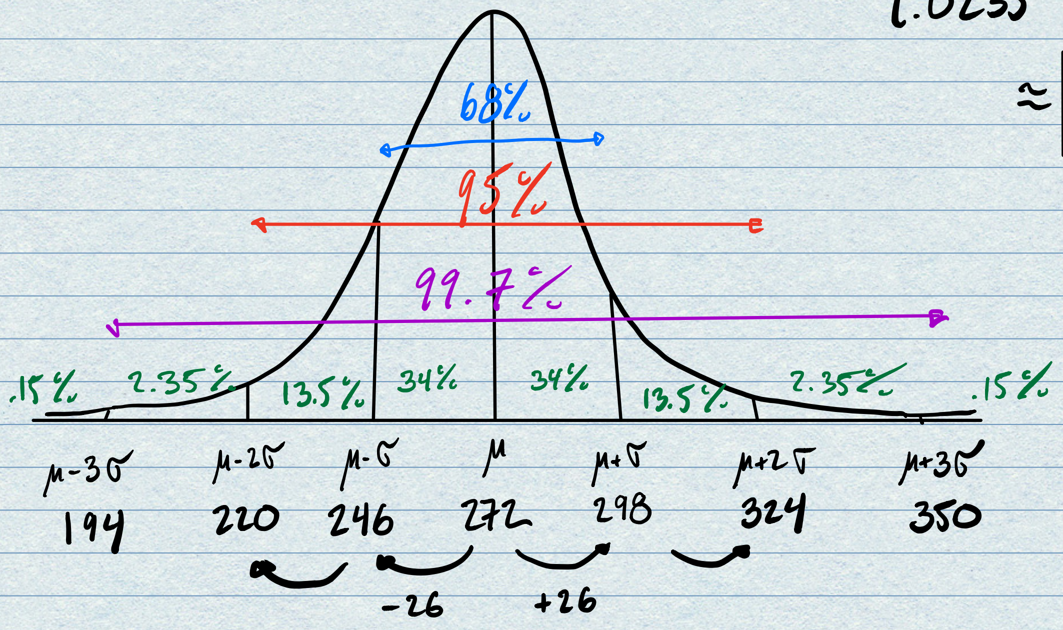
(b) BETWEEN 272 & 324 SEC? $\frac{228}{2} = \underline{114}$

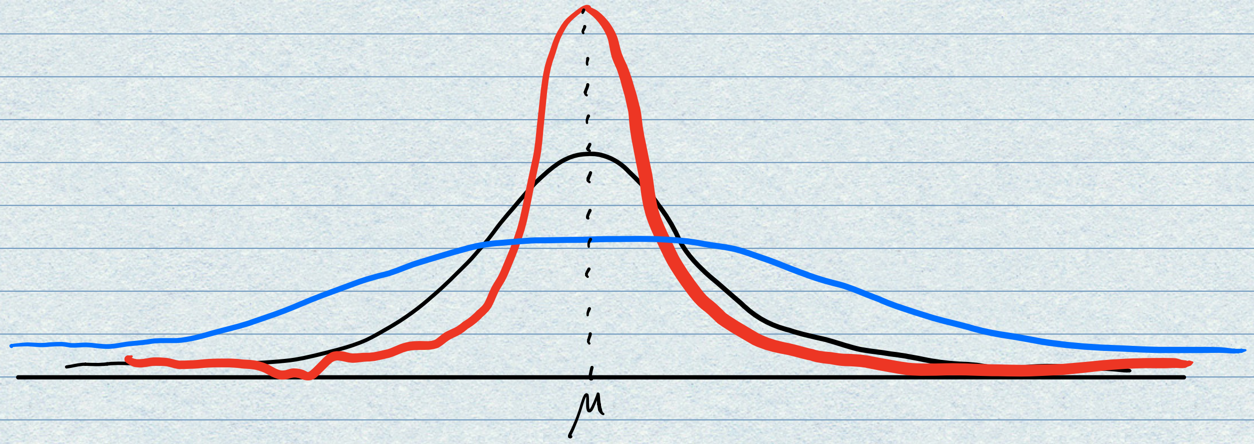
(c) BETWEEN 194 & 220 SEC?

2.35% of 240

$(.0235)(240) = \boxed{5.64}$

$\approx \boxed{6}$





3 DISTRIBUTION WITH THE SAME MEAN

SMALL σ

MEDIUM σ

LARGE σ

(σ MEASURES VARIATION "SPREAD")

COURSE: BASIC STATS ✓ → PROBABILITY → RETURN TO STATS.

CH. 4 PROBABILITY & PROBABILITY DISTRIBUTIONS

§4.2 EVENTS AND THE SAMPLE SPACE

INTRO TO SET THEORY

<u>Def</u>: A <u>SET</u> IS A COLLECTION OF OBJECTS.

NOTATION: OBJECTS LISTED INSIDE { } CURLY BRACKETS ARE CALLED ELEMENTS.

A SET IS DEFINED BY THE ELEMENTS THAT IT CONTAINS.
(ORDER OF ELEMENTS IS IRRELEVANT)

e.g. $\{a, e, i, o, u\}$ ← A SET WITH 5 ELEMENTS

$V = \{a, e, i, o, u\}$. $|V| = 5$

$\#V = n(V) = |V| =$ NUMBER OF ELEMENTS IN THE SET V

$= 5$

Def: A SET A IS A SUBSET OF A SET B IF EVERY ELEMENT IN A IS ALSO IN B.

e.g. $\{a, o, u\} \subset \{a, e, i, o, u\}$

e.g. $A = \{1, 3, 5\}$

$B = \{0, 1, 2, 3, 4, 5\}$

$C = \{1, 5\}$

$\#$ $<$

SETS $\subset$ $\subset=$

$8 > 5$ $5 < 8$

$$C \subset A \subset B \quad \left(B \supset A \supset C \right)$$

$$C \subset B \quad \left(B \supset C \right)$$

$A \not\subset C \subset B$?

↳ NOT A SUBSET

e.g. $S = \{a, b, c\}$

LIST ALL SUBSETS OF S.

8 SUBSETS

$\{a\}$

$\{b\}$

$\{c\}$

SETS THAT CONTAIN ONLY ONE ELEMENT ARE CALLED SINGLETONS.

$(1,2,3)$

FERMAT'S LAST THM : $S = \emptyset$

$$S = \{ (x,y,z) \mid x^3 + y^3 = z^3, \quad x,y,z \in \mathbb{N} \}$$

↳ POSITIVE WHOLE #'S

$\{a, b\}$

$\{b, c\}$

$\{a, c\}$

$\{\ \} = \emptyset$ THE EMPTY SET : THE SET THAT CONTAINS NO ELEMENTS.

$\{a, b, c\} = S$

↳ NOTE: S IS NOT A "PROPER" SUBSET OF S.

WE NEED SETS TO TALK ABOUT PROBABILITY...

Def: AN EXPERIMENT IS ANY PROCESS BY WHICH AN OBSERVATION IS MADE.

e.g. FLIP A COIN : OBSERVATION IS EITHER HEADS OR TAILS.

ROLLING A DIE / DICE

Def: THE SET OF ALL POSSIBLE OUTCOMES OF AN EXPERIMENT IS CALLED THE SAMPLE SPACE.

DENOTED (TYPICALLY) S OR Ω.

e.g. EXPERIMENT: FLIPPING A COIN TWICE

SAMPLE SPACE: $\{HH, HT, TH, TT\} = \Omega$

$|\Omega| = 4$

e.g. EXP: ROLL A DIE ONCE

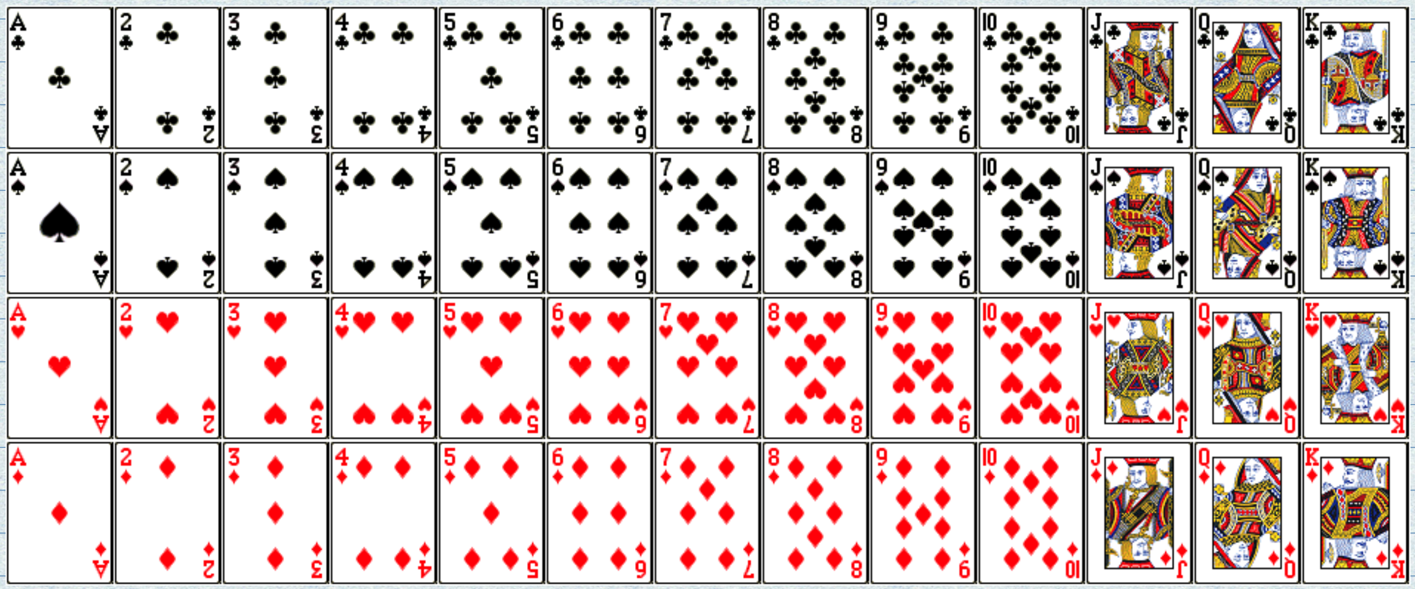
SAMPLE SPACE $\Omega = \{1, 2, 3, 4, 5, 6\}$

$|\Omega| = 6$

e.g. EXP: DEAL 3 CARDS FROM A STANDARD DECK OF 52

SAMPLE SPACE $\Omega = \{A\clubsuit J\heartsuit 3\diamondsuit, 9\clubsuit 8\heartsuit K\clubsuit, \ldots\}$

$|\Omega| = ?$ (WE WILL GET TO THIS LATER!)



<u>Def</u>: AN EVENT IS ANY SUBSET OF THE SAMPLE SPACE Ω.

A SIMPLE EVENT IS AN EVENT THAT CONTAINS ONLY ONE ELEMENT (POSSIBLE OUTCOME).

e.g. FLIPING COIN $\Omega = \{HH, HT, TH, TT\}$

EVENT: "AT LEAST ONE H"

$A = \{HH, HT, TH\}$ $A \subset \Omega$

A IS AN EVENT

$\{HH\} \leftarrow$ SIMPLE EVENT

SO IS $\{TH\}$, SO IS $\{TT\}$.