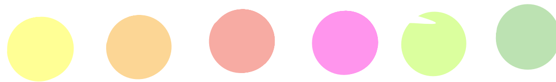
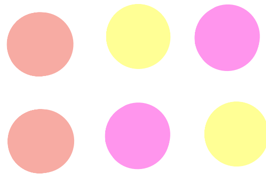
§4.4 Useful Counting Rules (cont'd)

Permutation: n objects 

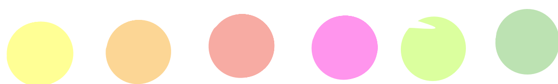
choose r objects, order matters.



(is DIFFERENT)

\# ways to do this is P^n_r (notation)

$$\boxed{P^n_r = \frac{n!}{(n-r)!}} = n(n-1)(n-2)\cdots(n-r+1)$$

e.g. 

$$P^6_3 = 6 \cdot 5 \cdot 4 = 120$$

$$\frac{6!}{(6-3)!} = \frac{6!}{3!}$$

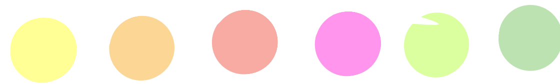
Calculator: P^n_r

Type n $\boxed{\text{MATH}} \rightarrow \boxed{\text{PRB}} \rightarrow \boxed{nPr} \rightarrow \boxed{\text{ENTER}}$

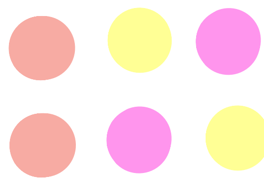
r $\boxed{\text{ENTER}}$

COMBINATIONS:

GIVEN n OBJECTS



CHOOSE r OBJECTS (ORDER DOES NOT MATTER)



(IS SAME)

\# WAYS TO DO THIS : C^n_r

$$\boxed{C^n_r = \frac{n!}{r!(n-r)!}} = \frac{n(n-1)(n-2)\cdots(n-r+1)}{r!}$$

CALCULATOR : C^n_r

TYPE n $\boxed{\text{MATH}} \rightarrow \boxed{\text{PRB}} \rightarrow \boxed{nCr} \rightarrow \boxed{\text{ENTER}}$

r $\boxed{\text{ENTER}}$

NOTE: GIVEN n OBJECTS, # WAYS TO ARRANGE ALL n OBJECTS IS $P^n_n = \frac{n!}{(n-n)!} = \frac{n!}{0!} = n!$

$$\frac{n}{1} \times \frac{n-1}{2} \times \frac{n-2}{3} \cdots \times \frac{1}{n} = n!$$

SO $0! = 1$

$n!$:= # WAYS TO ARRANGE n OBJECTS

$0!$:= # WAYS TO ARRANGE 0 OBJECTS $= 1$

4.29 A Card Game Three students are playing a card game. They decide to choose the first person to play by each selecting a card from the 52-card deck and looking for the highest card in value and suit. They rank the suits from lowest to highest: clubs, diamonds, hearts, and spades.

a. If the card is replaced in the deck after each student chooses, how many possible configurations of the three choices are possible?

b. How many configurations are there in which each student picks a different card?

c. What is the probability that all three students pick exactly the same card?

d. What is the probability that all three students pick different cards?

(a.) 3 STAGE EVENT

$$\underset{1^{st}}{\underline{52}} \times \underset{2^{nd}}{\underline{52}} \times \underset{3^{rd}}{\underline{52}}$$

$$= 52^3 = 140,608$$

ALL EQUALLY LIKELY.

(b.)

$$\underset{1^{st}}{\underline{52}} \times \underset{2^{nd}}{\underline{51}} \times \underset{3^{rd}}{\underline{50}} = P^{52}_{3} = \frac{52!}{49!}$$

$$= 132,600$$

(c.) # WAYS FOR 3 STUDENTS TO ALL PICK SAME CARD:

A♠ A♠ A♠
2♠ 2♠ 2♠
⋮
K♡ K♡ K♡

52 WAYS.

$$\underline{52} \times \underline{1} \times \underline{1} = 52$$

$$P\left(\begin{array}{c}\text{3 STUDENTS PICK}\\ \text{SAME CARD}\end{array}\right) = \frac{\text{\# WAYS TO ALL PICK SAME CARD}}{\text{\# POSSIBLE OUTCOMES}}$$

$$= \frac{52}{140,608} = 3.698 \text{ E } -4$$

$\rightarrow 3.698 \times 10^{-4}$ (MOVE DECIMAL 4 SPACES LEFT)

.0 0 0 3.698 = .0003698

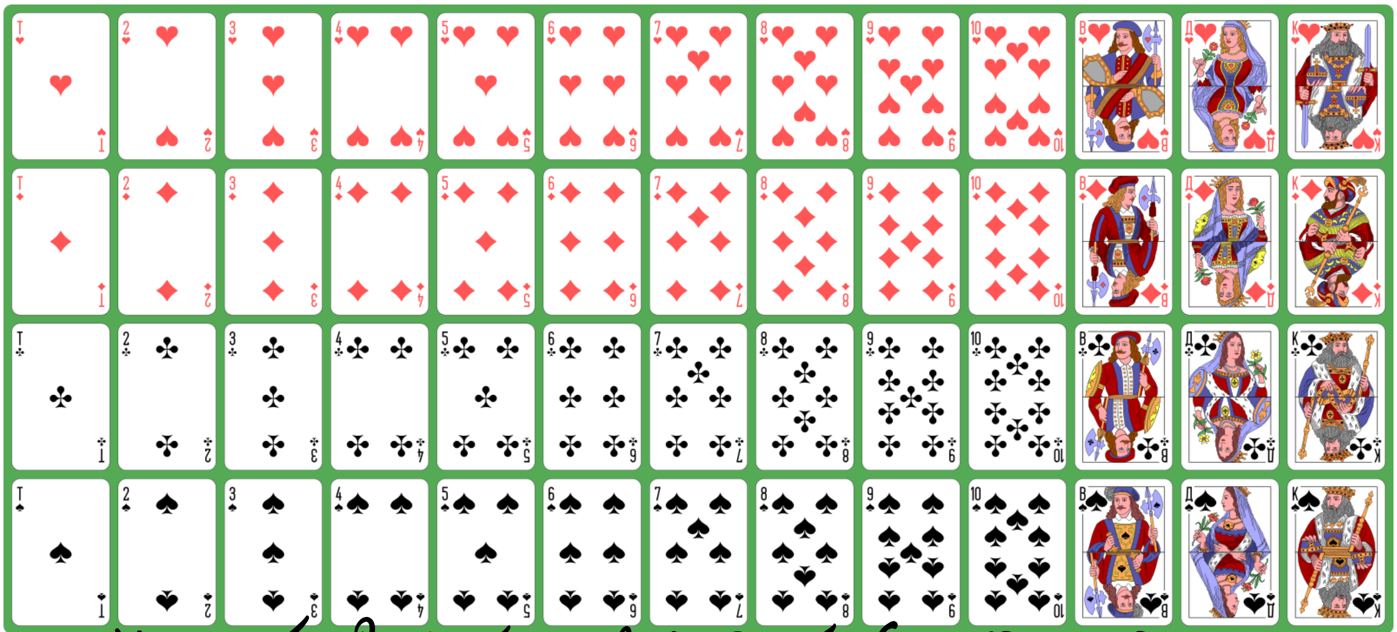
OR .03698 %

(d.) $P(\text{3 DIFFERENT CARDS}) = \dfrac{\text{\# WAYS TO GET 3 DIFFERENT CARDS}}{\text{TOTAL \# OF POSSIBLE OUTCOMES}}$

$= \dfrac{132{,}600}{140{,}608} = .9430$ OR 94.30%

V A L U E S

SUITS



ex. WHAT IS THE PROBABILITY OF BEING DEALT 5 CARDS AND HAVING A "FULL HOUSE"

= 2 OF A KIND (SAME VALUE)

AND 3 OF A KIND (SAME VALUE, DIFFERENT FROM THE 2 OF A KIND)

K-STAGE EVENT:

$\underline{13} \times \underline{C^4_2}$

PICK A VALUE FOR 2 OF A KIND

PICK THE 2 CARDS OF THAT VALUE

CANNOT BE THE SAME VALUE

SAME VALUE

$$\times \underset{\text{PICK A VALUE FOR 3 OF A KIND}}{\underline{12}} \times \underline{C^{4}_{3}}$$

$$C^{4}_{2} = \frac{4!}{(4-2)!\,2!} = \frac{4\cdot3\cdot2\cdot1}{2\cdot1\cdot2\cdot1}$$

$$\#\text{ WAYS TO BE DEALT FULL HOUSE} = 13 \times C^{4}_{2} \times 12 \times C^{4}_{3}$$

$$= 13 \times 6 \times 12 \times 4 = 3744$$

$$\#\text{ WAYS TO BE DEALT 5 CARDS} = C^{52}_{5} = 2{,}598{,}960$$

$$P(\text{FULL HOUSE}) = \frac{3744}{2{,}598{,}960} = \boxed{.001441}$$

<u>ex.</u> PROB. OF SELECTING 5 CARDS FROM A STANDARD DECK, ALL SAME SUIT (FLUSH).

$$P(\text{FLUSH}) = \frac{\#\text{ WAYS TO GET A FLUSH}}{\#\text{ POSSIBLE OUTCOMES.}}$$

$$= \frac{\overbrace{C^{4}_{1}}^{\text{PICK SUIT}} \times \overbrace{C^{13}_{5}}^{\text{PICK 5 CARDS (OF THAT SUIT)}}}{C^{52}_{5}}$$

$$= \frac{\dfrac{4!}{(4-1)!\,1!} \times \dfrac{13!}{(13-5)!\,5!}}{\dfrac{52!}{(52-5)!\,5!}}$$

$$= \frac{4 \times 1,287}{2,598,960} = \boxed{.001981}$$

ex. SUPPOSE A DRAWER CONTAINS 14 BATTERIES :

12 FRESH , 2 DEAD BATTERIES.

IF YOU SELECT 4 BATTERIES AT RANDOM ,

FIND PROBABILITY OF GETTING

(a) 0 DEAD BATTERIES
(b) 1 DEAD BATTERIES
(c) 2 DEAD BATTERIES

SUM OF PROB. = 1

(a) SIZE OF SAMPLE SPACE Ω

\# POSSIBLE OUTCOMES OF THIS EXPERIMENT

$$|\Omega| = C^{14}_{4} = \frac{14!}{10!\,4!} = 1001$$

ALL EQUALLY LIKELY !

\# POSSIBLE OUTCOMES WITH 0 DEAD BATTERIES ?
(4 FRESH BATTERIES)

$= C^{12}_{4}$ "12 choose 4"

$= \dfrac{12!}{8!\,4!} = 495$

$$P(0 \text{ dead batteries}) = \frac{C^{12}_{4}}{C^{14}_{4}} = \frac{495}{1001} = .4945$$

$$\frac{P^{12}_{4}}{P^{14}_{4}} = \frac{11{,}880}{24{,}024} = .4945$$

$\times 4!$ $\times 4!$

$$P^{n}_{r} = \frac{n!}{(n-r)!} \qquad C^{n}_{r} = \frac{n!}{(n-r)!\,r!}$$

$$P^{n}_{r} = C^{n}_{r}\,r! \qquad C^{n}_{r} = \frac{P^{n}_{r}}{r!}$$

4 objects: A, B, C, D

Choose 3

ABC	ACB	BAC	BCA	CAB	CBA
ABD	ADB	BAD	BDA	DAB	DBA
ACD	ADC	CAD	CDA	DAC	DCA
BCD	BDC	CBD	CDB	DBC	DCB

IF ORDER MATTERS: $P^4_3 = 24$

IF NOT: $\dfrac{P^4_3}{3!} = C^4_3$

ex. SUPPOSE A DRAWER CONTAINS 14 BATTERIES:
12 FRESH, 2 DEAD BATTERIES.

IF YOU SELECT 4 BATTERIES AT RANDOM,
FIND PROBABILITY OF GETTING
(a) 0 DEAD BATTERIES
(b) 1 DEAD BATTERIES
(c) 2 DEAD BATTERIES

SUM OF PROB. = 1

# DEAD BATT.	# FRESH BATTERIES	PROBABILITY
0	4	$\dfrac{C^{12}_4}{C^{14}_4} = .4945$
1	3	DEAD, FRESH: $\dfrac{C^2_1 \times C^{12}_3}{C^{14}_4} = .4396$
2	2	$\dfrac{C^2_2 \times C^{12}_2}{C^{14}_4} = .0659$

TOTAL $= .4945 + .4396 + .0659$
$= 1$ ✓

NOTE: $C^n_n = 1$ $\left(= \frac{n!}{(n-n)!\,n!} = \frac{n!}{0!\,n!} = 1 \right)$

$C^n_1 = n$ $\left(= \frac{n!}{(n-1)!\,1!} = n \right)$

SAME!

$C^n_{n-1} = n$

$C^n_r = C^n_{(n-r)}$ IN GENERAL.

ex. A RECORDING ARTIST RECORDS 17 TRACKS FOR A NEW ALBUM. THEY MUST CHOOSE 10 OF THESE SONGS & ORDER THEM TO CREATE AN ALBUM. HOW MANY WAYS TO DO THIS?

$P^{17}_{10} = \frac{17!}{7!} = 7.057 \text{ E } 10$

$= 7.057 \times 10^{10}$

$= 70,570,000,000$

4.32 **Poker II** Refer to Exercise 4.31. You have a poker hand containing four of a kind.

a. How many possible poker hands can be dealt?

b. In how many ways can you receive four cards of the same face value *and* one card from the other 48 available cards?

c. What is the probability of being dealt four of a kind?

$C^{13}_{1} \times C^{4}_{2}$

(b.)

PICK VALUE FOR 4 OF A KIND — PICK 4 CARD — PICK 5TH CARD

$$C^{13}_{1} \quad C^{4}_{4} \quad C^{48}_{1}$$

$$= 13 \times 1 \times 48$$

$$= 624$$