§6.4 Normal Approx. to Binomial Distr.

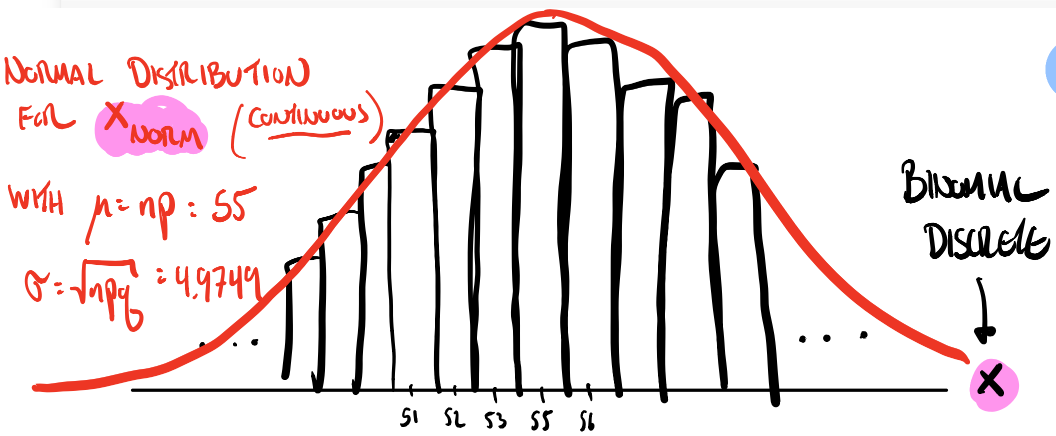
ex. Suppose 55% of MI voters support J.B.
We survey a random sample of 100 MI voters.
Let X = # of MI voters in sample that support J.B.
Find the probability that $X \geq 51$.

X is discrete. Possible values for X: $0, 1, 2, \ldots, 100$

X is binomial, $n = 100$ trials

$p = .55$, $q = .45$

$$P(X=k) = C^{n}_{k}\, p^{k} q^{n-k} = C^{100}_{k} (.55)^{k} (.45)^{100-k}$$



For Binomial R.V.

Mean $\mu = np = (100)(.55) = 55$

$\sigma = \sqrt{npq}$ (§5.2)

$= \sqrt{(100)(.55)(.45)}$

$= 4.9749$

$$P(X \geq 51) = \sum_{k=51}^{100} P(X=k)$$

WHAT IF WE TREAT X JUST LIKE X_{NORM}?

$$P(X \geq 51) \rightsquigarrow P(X_{NORM} \geq 51)$$

$$= P\left(Z \geq \frac{51 - (100)(.55)}{\sqrt{(100)(.55)(.45)}}\right) \qquad Z = \frac{x-\mu}{\sigma}$$

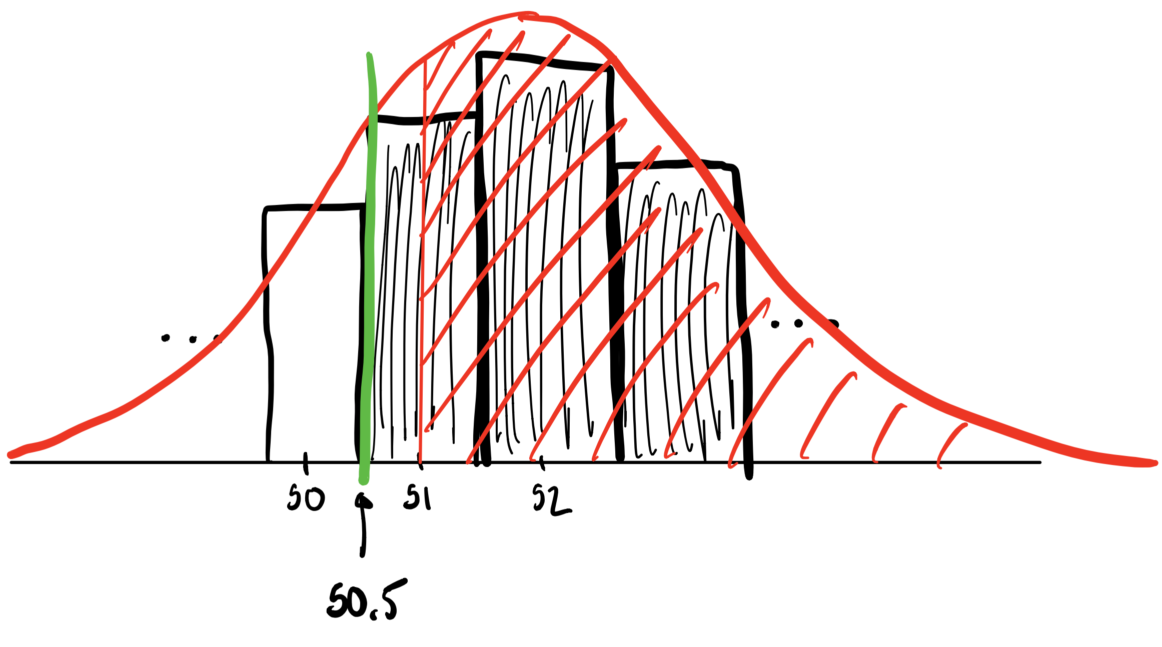
$$= P(Z \geq -.80)$$

$$= 1 - P(Z \leq -.80) = 1 - .2119 = .7881$$

	.00									
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776

ACTUAL ANSWER: $P(X \geq 51) = .817272$

$$\text{REL. ERROR} = \frac{\text{APPROX} - \text{ACTUAL}}{\text{ACTUAL}} \approx 4\%$$



USING 50.5 :

$$P(x \geq 51) \approx P(x_{NORM} \geq 50.5)$$

$$z = \frac{x - \mu}{\sigma}$$

$$z = \frac{x - np}{\sqrt{npq}}$$

$$P\left(z \geq \frac{50.5 - (100)(.55)}{\sqrt{(100)(.55)(.45)}}\right)$$

$$= P(z \geq -.90)$$

$$= 1 - P(z \leq -.90)$$

$$= 1 - .1841 = \boxed{.8159}$$

BETTER!

ACTUAL ANSWER: $P(x \geq 51) = .817272$

6.47 No Shows Airlines and hotels often grant reservations in excess of capacity to minimize losses due to no-shows. Suppose the records of a hotel show that, on the average, 10% of their prospective guests will not claim their reservation. If the hotel accepts 215 reservations and there are only 200 rooms in the hotel, what is the probability that all guests who arrive to claim a room will receive one?

$$C^{215}_{200} (.9)^{200} (.1)^{15}$$

PROB. EXACTLY 200 SHOW UP

$$P(x = k) = C^{n}_{k} p^{k} q^{n-k}$$

$n = 215$ TRIALS

$p = .9$ SHOWS UP

$q = .1$ NO-SHOW

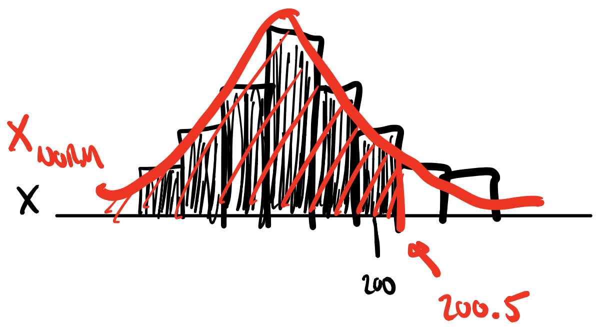
x = # GUESTS WHO SHOW UP

x IS BINOMIAL R.V.

FIND $P(x \leq 200)$

X is "NORMAL-ISH" with mean $\mu = np = (215)(.9) =$

& stand dev $\sigma = \sqrt{npq} = \sqrt{(215)(.9)(.1)}$



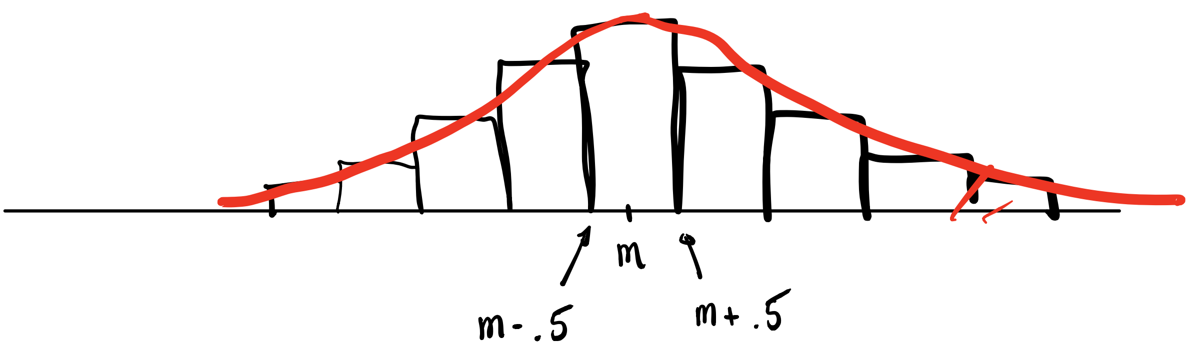
$$P(X \le 200) \approx P(X_{NORM} \le 200.5)$$

CORRECTING FOR CONTINUITY

APPROX A DISCRETE R.V. WITH CONTINUOUS R.V.

$$= P\left(Z \le \frac{200.5 - (215)(.9)}{\sqrt{(215)(.9)(.1)}}\right) = P(Z \le 1.59) = .9441$$

1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767



$$P(X \ge m) \approx P(X_{NORM} \ge m - .5)$$
$$P(X \le m) \approx P(X_{NORM} \le m + .5)$$

$$P(X > m) \approx P(X_{NORM} \ge m + .5)$$
$$P(X < m) \approx P(X_{NORM} \le m - .5)$$

6.50 The Rh Factor In a certain population, 15% of the people have Rh-negative blood. A blood bank serving this population receives 92 blood donors on a particular day.

a. What is the probability that 10 or fewer are Rh-negative?

b. What is the probability that 15 to 20 (inclusive) of the donors are Rh-negative?

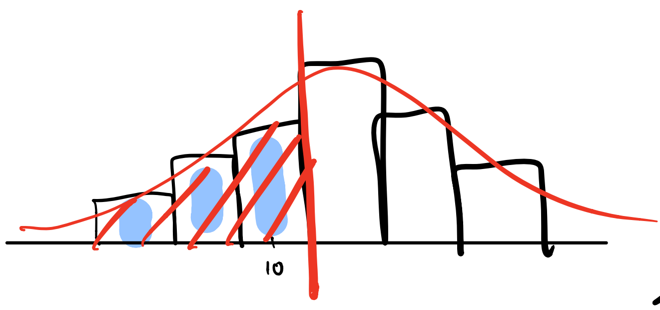
c. What is the probability that more than 80 of the donors are Rh-positive?

ASSUME RESULT OF EACH TEST IS INDEPENDENT.

$n = 92$ TRIALS $\qquad x = \#$ RH-NEG. $\qquad$ STATS FOR x:

$p = .15$ $\qquad$ MEAN $\mu = np = (92)(.15)$

$q = .85$ $\qquad$ STAND. DEV. $\sigma = \sqrt{npq} = \sqrt{(92)(.15)(.85)}$



$$z = \frac{x-\mu}{\sigma} = \frac{x - np}{\sqrt{npq}}$$

$$P(x \le 10) \approx P(x_{\text{NORM}} \le 10.5) = P\left(z \le \frac{10.5 - (92)(.15)}{\sqrt{(92)(.15)(.85)}}\right)$$

CORRECTING FOR CONTINUITY

$$= P(z \le -.96) = .1685$$

	.00	.01	.02				.06			
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
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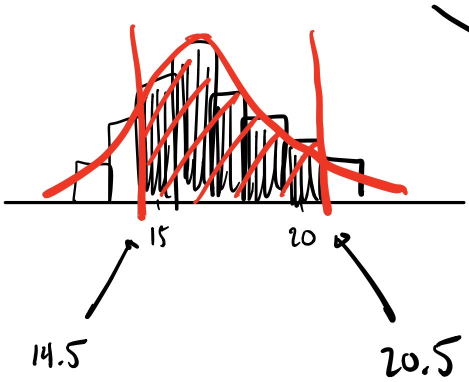
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776

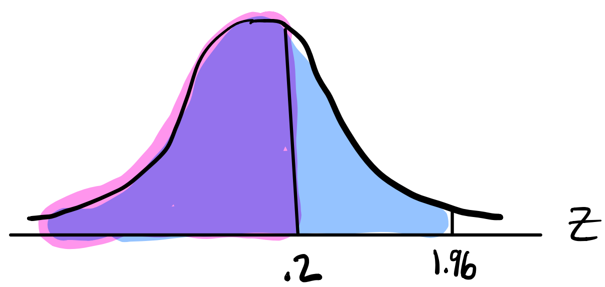
$$P(15 \le x \le 20) \approx P(14.5 \le X_{norm} \le 20.5)$$

CORRECT FOR CONTINUITY



$$P\left(\frac{14.5-(92)(.15)}{\sqrt{(92)(.15)(.85)}} \le z \le \frac{20.5-(92)(.15)}{\sqrt{(92)(.15)(.85)}}\right)$$

$$= P(.20 \le z \le 1.96)$$



$$= P(z \le 1.96) - P(z \le .2)$$

$$= .9750 - .5793 = .3957$$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
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1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767

RULE OF THUMB

The normal approximation to the binomial probabilities will be adequate if both

$np > 5$ and $nq > 5$ ← DISTRIBUTION FOR X IS SKEWED