Name: *ANSWER KEY* 10/29/2018

Math 173 Introduction to Probability and Statistics **Quiz 3**

1. Suppose you own a chicken that lays eggs, and each egg has a 5% chance of having a double-yoke. You gather 12 eggs laid by this chicken and count x the number of eggs with double-yokes.

   (a) (4 points) What is the expected value $E[x]$ (i.e. mean μ) for x?

   NOTE: X IS A BINOMIAL RANDOM VARIABLE WITH $n = 12$, $p = .05$, $q = .95$

   $$E[x] = \mu = np = (12)(.05) = \boxed{.6}$$

   (b) (4 points) What is the standard deviation σ for x?

   $$\sigma = \sqrt{npq} = \sqrt{(12)(.05)(.95)} = \boxed{.7550}$$

   (c) (8 points) What is the probability that you get more than one egg with a double-yoke?

   $$P(x > 1) = 1 - P(x \leq 1) = 1 - .882 = \boxed{.118}$$

   LOOK THIS UP IN TABLE 1,

   OR $P(x \leq 1) = P(x = 0) + P(x = 1)$

   $$= C^{12}_{0} (.05)^0 (.95)^{12} + C^{12}_{1} (.05)^1 (.95)^{11}$$

   $$= .5407 + .3413 = .8820$$

2. (8 points) Your desk drawer contains 5 new batteries and 3 old batteries. You reach in a take 4 batteries. What is the probability that you took exactly 2 new batteries and 2 old batteries?

   LET x = # NEW BATTERIES. x IS HYPERGEOMETRIC DISCRETE RANDOM VARIABLE

   WITH $N = 8$

   $M = 5$

   $n = 4$

   $$P(x = 2) = \frac{C^5_2 \, C^3_2}{C^8_4} = \frac{10 \cdot 3}{70}$$

   $$= \boxed{\frac{3}{7} \approx .4286}$$

3. A normal random variable x has mean $\mu = 36$ and standard deviation $\sigma = 4$. Find each of the following probabilities.

(a) (4 points) $P(x \le 30)$

$$z = \frac{x-\mu}{\sigma} = \frac{30-36}{4} = -1.5 \quad , \quad P(x \le 30) = P(z \le -1.5) = \boxed{.0668}$$

(b) (4 points) $P(x \ge 44)$

$$z = \frac{44-36}{4} = 2 \quad , \quad P(x \ge 44) = P(z \ge 2) = 1 - P(z \le 2) = 1 - .9772 = \boxed{.0228}$$

(c) (4 points) $P(30 < x < 44)$

$$P(30 < x < 44) = P(-1.5 < z < 2) = P(z \le 2) - P(z \le -1.5) = .9772 - .0668$$
$$= \boxed{.9104}$$

4. Airlines and hotels often grant reservations in excess of capacity to minimize losses due to no-shows. Suppose the records of a hotel show that, on average, 10% of their prospective guests will not claim their reservation. Suppose the hotel accepts 215 reservations and let x equal the number of those reservations that arrive to claim their room.

(a) (4 points) What "rule of thumb" allows us to approximate the binomial random variable x with a normal distribution?

$\boxed{np > 5}$ $\quad$ $\boxed{nq > 5}$

$(215)(.9) = 193.5$ ✓ $\quad$ $(215)(.1) = 21.5 > 5$ ✓

(b) (8 points) If there are only 200 rooms in the hotel, what is the probability that all guests who arrive to claim a room will receive one?

NOTE : SINCE X = # SUCCESSES = # GUESTS WHO ARRIVE TO CLAIM THEIR ROOM

WE HAVE $p = .9$, $q = .1$

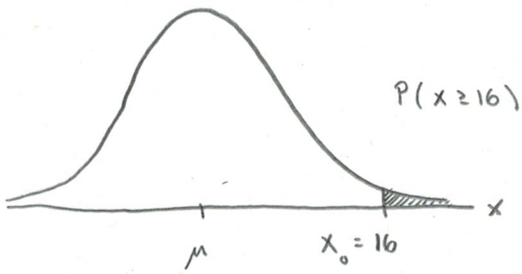
$$P_{BIN}(x \le 200) \approx P_{NORM}(x \le 200.5) = \ldots$$

$$\mu = np = (215)(.9) = 193.5 \quad , \quad \sigma = \sqrt{npq} = \sqrt{(215)(.9)(.1)} \approx 4.3989$$

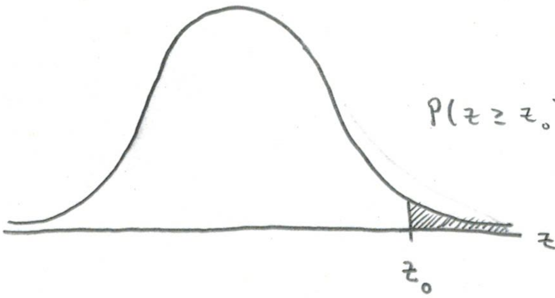
$$z = \frac{200.5 - 193.5}{4.3989} = 1.5913$$

$$\ldots = P(z \le 1.59) = \boxed{.9441}$$

5. (8 points) A peanut farmer owns a peanut dispenser that can be set to dispense peanuts in amounts that are normally distributed, with mean μ and standard deviation $\sigma = 0.25$ ounces. If the farmer wishes to use the machine to fill containers that hold 16 ounces of peanuts and wants to overfill only 2% of the containers, at what value of μ should the farmer set the peanut dispenser?



$P(x \geq 16) = .02$



$P(z \geq z_0) = .02 \Rightarrow P(z \leq z_0) = .98$

$z_0 \approx 2.05$

(REVERSE LOOK-UP IN TABLE)

$$z_0 = \frac{x_0 - \mu}{\sigma} \longrightarrow 2.05 = \frac{16 - \mu}{.25}$$

$$(2.05)(.25) = 16 - \mu$$

$$\mu = 16 - (2.05)(.25)$$

$$\mu = 16 - .5125$$

$$\mu = \boxed{15.4875}$$