MATH 173

QUIZ 3

6/26/2017

NAME: *ANSWER KEY*

1. AN EXPERIMENT CAN RESULT IN EVENTS A AND B WITH THE FOLLOWING PROBABILITIES.

	A	A^c	TOTAL
B	.45	.15	.6
B^c	.35	.05	.4
TOTAL	.8	.2	1

(a) FIND $P(A|B)$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{(.45)}{(.6)} = \boxed{.75}$$

(b) ARE A AND B INDEPENDENT? WHY / WHY NOT?

$\boxed{\text{No}}$ BECAUSE $P(A \cap B) \neq P(A)P(B)$

$.45 \neq (.8)(.6) = .48$

(c) ARE A AND B MUTUALLY EXCLUSIVE? WHY / WHY NOT?

$\boxed{\text{No}}$ BECAUSE $P(A \cap B) \neq 0$

2. PLAYER A HAS ENTERED A GOLF TOURNAMENT BUT IT IS NOT CERTAIN WHETHER PLAYER B WILL ENTER. PLAYER A HAS PROBABILITY $\frac{1}{6}$ OF WINNING THE TOURNAMENT IF PLAYER B ENTERS AND PROBABILITY $\frac{3}{4}$ OF WINNING IF PLAYER B DOES NOT ENTER THE TOURNAMENT. IF THE PROBABILITY THAT B ENTERS IS $\frac{1}{3}$. FIND THE PROBABILITY THAT PLAER A WINS THE TOURNAMENT.

LET A = EVENT THAT PLAYER A WINS THE TOURNAMENT

B = EVENT THAT PLAYER B ENTERS THE TOURNAMENT.

GIVEN: $P(A \mid B) = \frac{1}{6}$ FIND: $P(A)$

$P(A \mid B^c) = \frac{3}{4}$

$P(B) = \frac{1}{3}$

BY LAW OF TOTAL PROBABILITY:

$$P(A) = P(B)P(A \mid B) + P(B^c)P(A \mid B^c)$$

$$= \left(\frac{1}{3}\right)\left(\frac{1}{6}\right) + \left(\frac{2}{3}\right)\left(\frac{3}{4}\right)$$

$$= \frac{1}{18} + \frac{1}{2} = \frac{1+9}{18} = \frac{10}{18} = \boxed{\frac{5}{9} \text{ OR } .5556}$$

3. A SMOKE-DETECTOR SYSTEM USES TWO DEVICES, A AND B. IF SMOKE IS PRESENT, THE PROBABILITY THAT IT WILL BE DETECTED BY DEVICE A IS .95; BY DEVICE B, .98; AND BY BOTH DEVICES, .94. IF SMOKE IS PRESENT, FIND THE PROBABILITY THAT SMOKE WILL BE DETECTED BY DEVICE A OR DEVICE B OR BOTH DEVICES.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= .95 + .98 - .94$$

$$= \boxed{.99}$$

4. A WORKER-OPERATED MACHINE PRODUCES A DEFECTIVE ITEM WITH PROBABILITY .01 IF THE WORKER FOLLOWS THE MACHINE'S OPERATING INSTRUCTIONS EXACTLY, AND WITH PROBABILITY .03 IF HE DOES NOT. IF THE WORKER FOLLOWS THE INSTRUCTIONS 90% OF THE TIME, WHAT PROPORTION OF ALL ITEMS PRODUCED BY THE MACHINE WILL BE DEFECTIVE?

LET D = EVENT THAT MACHINE PRODUCES DEFECTIVE ITEM

I = EVENT THAT WORKER FOLLOWS INSTRUCTIONS EXACTLY.

GIVEN: $P(D|I) = .01$ FIND: $P(D)$

$P(D|I^c) = .03$ (USE LAW OF TOTAL PROBABILITY)

$P(I) = .9$

$$P(D) = P(I)P(D|I) + P(I^c)P(D|I^c)$$

$$= (.9)(.01) + (.1)(.03) = .009 + .003 = \boxed{.012}$$

5. MEDICAL CASE HISTORIES INDICATE THAT DIFFERENT ILLNESSES MAY PRODUCE IDENTICAL SYMPTOMS. SUPPOSE A PARTICULAR SET OF SYMPTOMS, WHICH WE DENOTE AS EVENT H, OCCURS ONLY WHEN ANY ONE OF THREE ILLNESSES - A, B, OR C - OCCURS. (FOR THE SAKE OF SIMPLICITY, WE WILL ASSUME THAT ILLNESSES A, B, AND C ARE MUTUALLY EXCLUSIVE.) STUDIES SHOW THESE PROBABILITIES OF GETTING THE THREE ILLNESSES:

$P(A) = .01 \qquad P(B) = .005 \qquad P(C) = .02.$

THE PROBABILITIES OF DEVELOPING THE SYMPTOMS H, GIVEN A SPECIFIC ILLNESS, ARE

$P(H|A) = .90 \qquad P(H|B) = .95 \qquad P(H|C) = .75.$

ASSUMING THAT AN ILL PERSON SHOWS THE SYMPTOMS H, WHAT IS THE PROBABILITY THAT THE PERSON HAS ILLNESS C?

$$P(C|H) \underset{\substack{\uparrow \\ \text{BAYES' RULE}}}{=} \frac{P(C)P(H|C)}{P(H)} \underset{\substack{\uparrow \\ \text{LAW OF TOTAL PROB.}}}{=} \frac{P(C)P(H|C)}{P(A)P(H|A)+P(B)P(H|B)+P(C)P(H|C)}$$

$$= \frac{(.02)(.75)}{(.01)(.9)+(.005)(.95)+(.02)(.75)} = \boxed{.5217}$$