

Name: *Answer Key*

Math 17300-CD Introduction to Probability and Statistics

5/8/2019

Quiz 4

Please put away all papers and electronic devices except a calculator. Show enough work that it is clear how you arrived at your answer. Correct answers with no work shown will not receive full credit. Answers that are decimals should be accurate to 4 decimal places. Box/circle your final answers. Good luck!

1. (8 points) Assume that 14% of all people who follow A on Instagram will click on an advertisement for B when it appears in their feed. Suppose you pay Instagram to show this advertisement to 500 randomly selected followers of A . Use a normal approximation to determine the probability that 60 or more of these selected followers will click on the advertisement for B .

$$np = (500)(.14) = 70 \geq 5$$

$$nq = (500)(.86) = 430 \geq 5$$

$$\therefore X_{\text{BIN}} \approx X_{\text{NORM}}$$

$$P(X_{\text{BIN}} \geq 60) \approx P(X_{\text{NORM}} \geq 59.5)$$

$$\mu = np = (500)(.14) = 70$$

$$\sigma = \sqrt{npq} = \sqrt{(500)(.14)(.86)} = \sqrt{60.2}$$

$$\approx 7.7589$$

$$z = \frac{x - \mu}{\sigma} = \frac{59.5 - 70}{\sqrt{60.2}} \approx -1.3533$$

$$\rightarrow P(z \geq -1.3533) = 1 - P(z \leq -1.3533)$$

$$\approx 1 - .0880 = \boxed{.9120}$$

$$\left(\text{TABLE : } 1 - .0885 = .9115 \right)$$

2. (8 points) Suppose that for all watermelons grown on a particular farm, the mean weight is $\mu = 13$ lbs and the standard deviation is $\sigma = 2.4$ lbs. If you order 100 watermelons from this farm, what is the probability that the total weight of the watermelons you receive is less than 1280 lbs? (Equivalently, what is the probability that the mean weight of the watermelons you receive is less than 12.8 lbs?)

$n \geq 30 \Rightarrow \sum x$ & \bar{x} ARE BOTH APPROXIMATELY NORMALLY DISTRIBUTED.

$$\begin{aligned} & \text{MEAN } \mu = 13, \quad \text{S.E.} = \frac{\sigma}{\sqrt{n}} = \frac{2.4}{\sqrt{100}} = .24 \\ & \text{MEAN } n\mu = 1300, \quad \text{S.E.} = \sigma\sqrt{n} = (2.4)\sqrt{100} = 24 \end{aligned}$$

$$\begin{aligned} P(\sum x < 1280) &= P\left(z < \frac{1280 - 1300}{24}\right) = \\ P(\bar{x} < 12.8) &= P\left(z < \frac{12.8 - 13}{.24}\right) = \end{aligned} \left\{ \begin{aligned} & P(z < -.8333) \approx \boxed{.2023} \\ & \text{(TABLE: } \approx .2033) \end{aligned} \right.$$

3. (8 points) A random sample of $n = 900$ observations from a binomial population produced $x = 655$ successes. Estimate the population proportion p and calculate the margin of error. (Please note, your estimate is a point estimate, and the margin of error is $1.96 \times \text{S.E.}$)

$$p \approx \hat{p} = \frac{x}{n} = \frac{655}{900} \approx \boxed{.7278}$$

$$\text{MARGIN OF ERROR} = 1.96 \sqrt{\frac{\hat{p}\hat{q}}{n}} = 1.96 \sqrt{\frac{(\frac{655}{900})(\frac{245}{900})}{900}}$$

$$\approx \boxed{.0291}$$

$$\left(p \approx 72.78\% \pm 2.91\% \right)$$

4. (8 points) Acid rain, caused by the reaction of certain air pollutants with rainwater, is a growing problem in the United States. Pure rain falling through clean air registers a pH value of 5.7 (pH is a measure of acidity: 0 is acid, 14 is alkaline). Suppose water samples from 40 rainfalls are analyzed for pH, and \bar{x} and s are 3.7 and .5, respectively. Find a 99% confidence interval for the mean pH in rainfall.

$$\mu \approx \bar{x} \pm z_{\alpha/2} \text{ S.E.}, \quad \bar{x} = 3.7, \quad z_{\alpha/2} = 2.58,$$

$$\text{S.E.} = \frac{s}{\sqrt{n}} \approx \frac{.5}{\sqrt{40}} = \frac{.5}{\sqrt{40}} \approx .0791$$

$$\begin{array}{c} \underbrace{- .2041} \\ [3.7 - 2.58 (.0791), 3.7 + 2.58 (.0791)] \\ \underbrace{+ .2041} \end{array}$$

$$[3.4959, 3.9041]$$

5. (8 points) Suppose you run a small company and are interested in knowing what proportion of your customers are satisfied with your service. So you contact 240 randomly selected customers and ask them to answer some survey questions. You find that 85% of these customers are satisfied with your service. Create a 99% confidence interval for the proportion of *all* your customers that are satisfied with your service.

$$p \approx \hat{p} \pm z_{\alpha/2} \text{ S.E.}, \quad \hat{p} = .85, \quad z_{\alpha/2} = 2.58,$$

$$\text{S.E.} \approx \sqrt{\frac{\hat{p}\hat{q}}{n}} = \sqrt{\frac{(.85)(.15)}{240}} \approx .0230$$

$$\begin{array}{c} \underbrace{- .0593} \\ [.85 - 2.58 (.0230), .85 + 2.58 (.0230)] \\ \underbrace{+ .0593} \end{array}$$

$$[.7907, .9093]$$