§2.4 ON THE PRACTICAL SIGNIFICANCE OF STANDARD DEVIATION

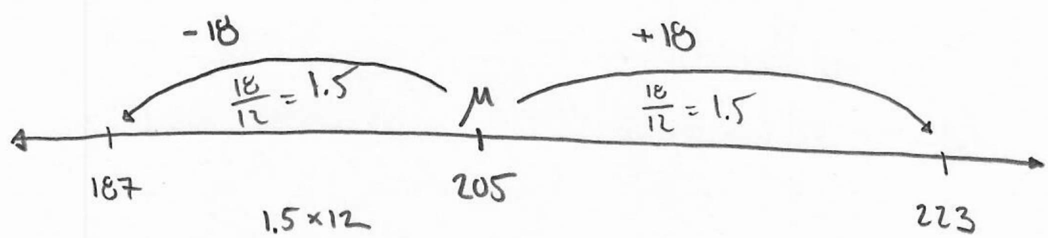
CHEVYCHEF'S THM:

GIVEN ANY NUMBER $k \geq 1$ AND ANY SET OF n MEASUREMENTS, AT LEAST $\left[1 - \frac{1}{k^2}\right]$ OF THE MEASUREMENT LIE <u>WITHIN k STANDARD DEVIATIONS OF THE MEAN</u>.

ex. YOU HAVE PLAYLIST OF 50 SONGS. AVERAGE SONG LENGTH IS 205 SEC. WITH STAND. DEV. 12 SEC. USE C.T. TO OBTAIN A MINIMUM NUMBER OF SONGS WITH LENGTH BETWEEN 187 SEC & 223 SEC. "WITHIN 1.5 S.D. OF THE MEAN."

$$\underset{\uparrow\, k}{1.5}$$

$N = 50, \quad \mu = 205, \quad \sigma = 12.$



$$\underset{1.5 \times 12}{\downarrow}$$

$187 = 205 - 18 = \mu - 1.5\boxed{\sigma}$ 1.5 S.D. BELOW MEAN

$223 = 205 + 18 = \mu + 1.5\boxed{\sigma}$ 1.5 S.D. ABOVE MEAN

$[187, 223]$

$$\left[\begin{array}{cc} 205 - 18 & , \quad 205 + 18 \\ \uparrow & \uparrow \\ \mu & \mu \end{array}\right]$$

$$\frac{18}{\sigma} = \frac{18}{12} = 1.5$$

$$\Rightarrow 18 = 1.5(12)$$

$$18 = 1.5\sigma$$

$$\underbrace{[\mu - 1.5\sigma, \quad \mu + 1.5\sigma]}$$

#'S THAT LIE WITHIN $\underset{K}{1.5}$ STAND. DEV. OF μ

AT LEAST $\left[1 - \frac{1}{1.5^2}\right]$ OF MEAS. LIE WITHIN 1.5 STAND. DEV. OF THE MEAN.

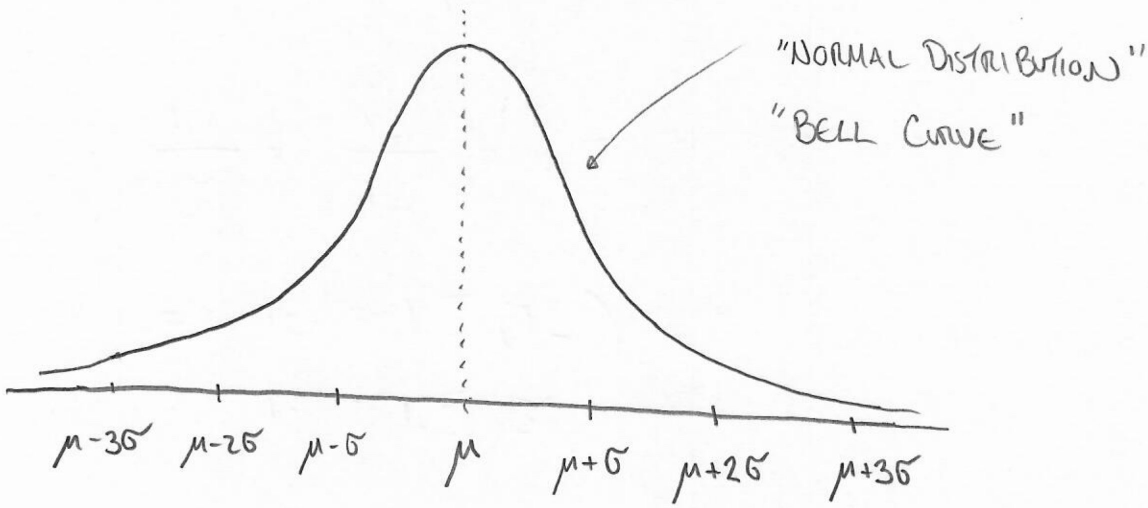
$$1 - \frac{1}{1.5^2} = 1 - \frac{1}{2.25} = .5555 \text{ or } 55.55\%$$

$.5555$ OF 50 : $(.5555)(50) = 27.7778$

ROUND UP: $\boxed{28 \text{ SONGS}}$

CHEVYCHEV'S THEOREM APPLIES TO ALL DISTRIBUTIONS.

* IF WE ASSUME THAT A DISTRIBUTION IS SYMMETRIC & UNIMODAL, THEN WE CAN BE MORE SPECIFIC

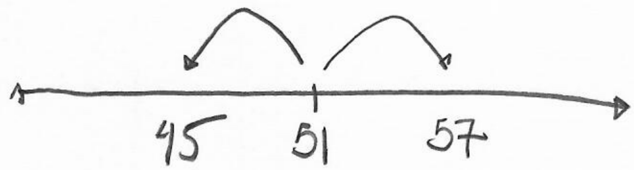


EMPERICAL RULE:

- APPROXIMATELY 68% OF MEASUREMENTS LIE WITHIN 1 STANDARD DEVIATION OF THE MEAN.
- APPROXIMATELY 95% OF MEASUREMENTS LIE WITHIN 2 STANDARD DEVIATIONS OF THE MEAN
- APPROXIMATELY 99.7% OF MEASUREMENTS LIE WITHIN 3 STANDARD DEVIATIONS OF THE MEAN.

ex. SUPPOSE $\mu = 51$

$\sigma = 6$.



WHAT NUMBERS LIE WITHIN 1 STANDARD DEVIATION OF THE MEAN?

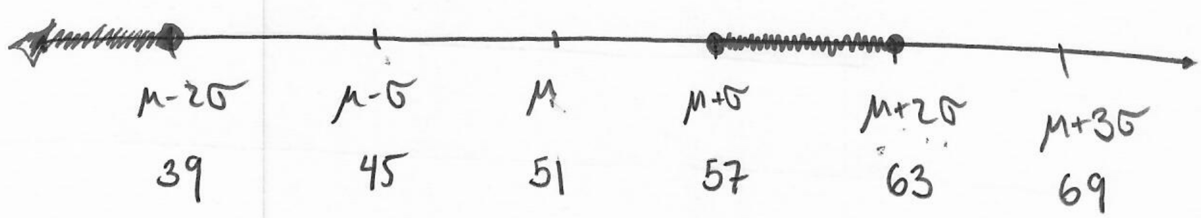
$$[\mu - 1\sigma \,,\, \mu + 1\sigma]$$

$$[51 - 1(6) \,,\, 51 + 1(6)]$$

$$[45, 57]$$ ⟵ INTERVAL NOTATION

$45 \le X \le 57$

WHAT NUMBERS LIE BETWEEN 1 S.D. ABOVE THE MEAN & 2 SD. ABOVE THE MEAN? $[57, 63]$



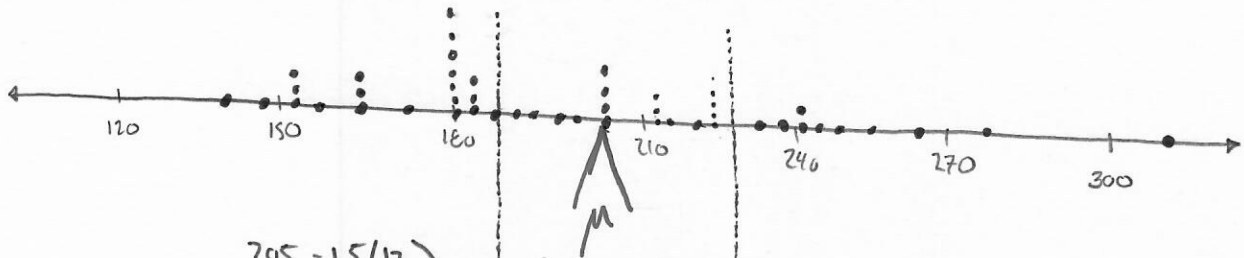
WHAT NUMBERS LIE AT LEAST 2 S.D. BELOW THE MEAN? $X \le 39$ $(-\infty, 39]$

$N = 50$

$\mu = 205$

$\sigma = 12$

• = 1 SONG LENGTH



$205 - 1.5(12)$ #'S WITHIN 1.5 σ OF μ $205 + 1.5(12)$

$\mu - 1.5\sigma = 187$ $223 = \mu + 1.5\sigma$

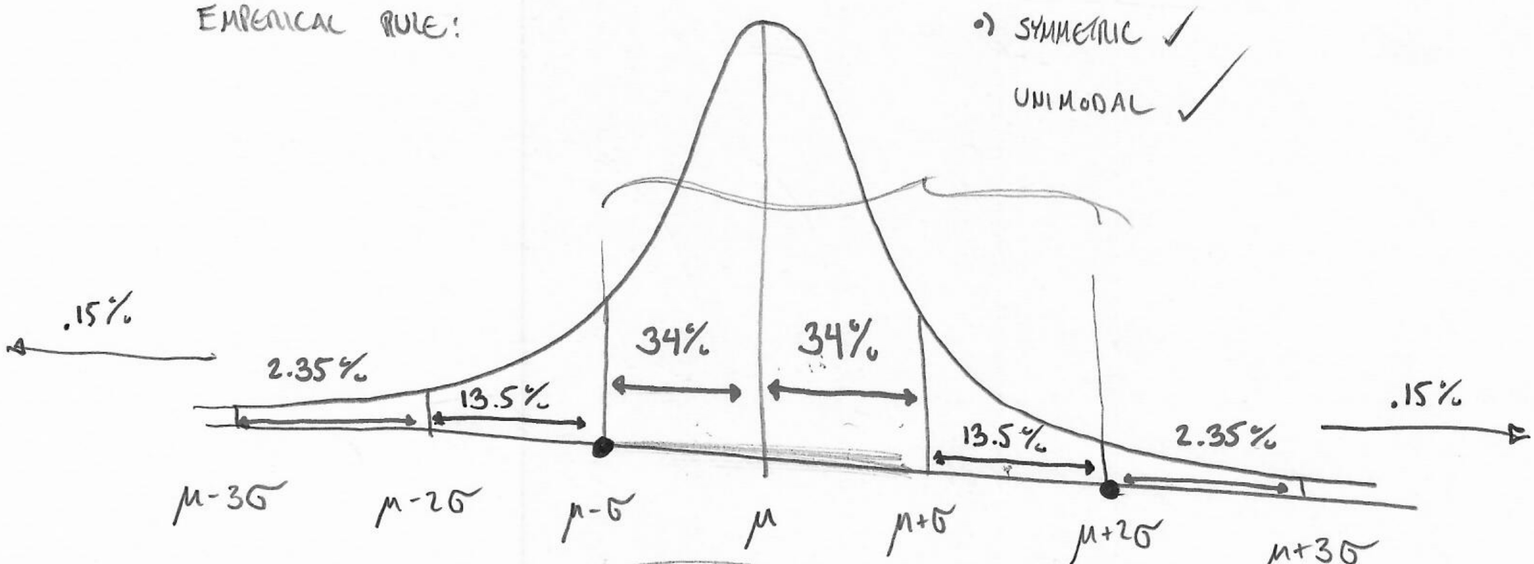
AT LEAST $1 - \frac{1}{K^2}$ OF MEASUREMENT

LIE WITHIN K STAND. DEV. OF MEAN

EMPERICAL RULE:

• SYMMETRIC ✓

UNIMODAL ✓



$\approx 68\%$

$\approx 95\% = 68\% + 27\%$

$\approx 99.7\% = 95\% + 4.7\%$

SPLIT BETWEEN

RIGHT & LEFT

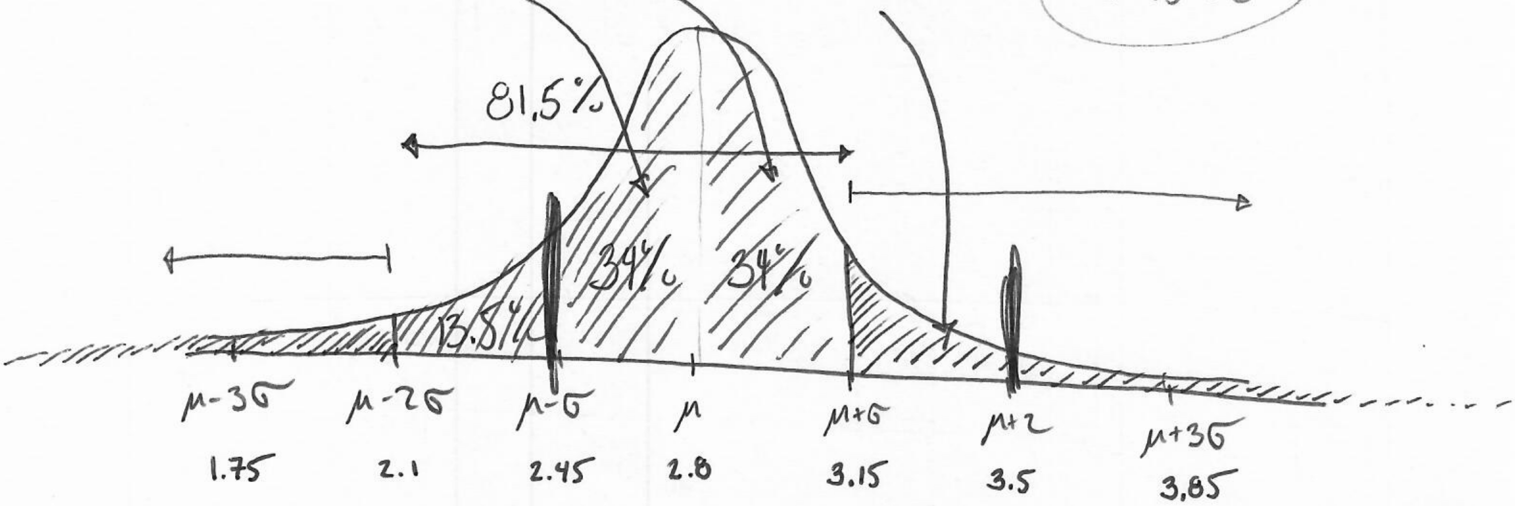
ex. SUPPOSE THE THICKNESS OF SLICED HAM AT A PROCESSING PLANT IS NORMALLY DISTRIBUTED ($\underbrace{\text{i.e. SYMMETRIC \& UNIMODAL}}_{\text{EMPERICAL RULE APPLIES.}}$)

SUPPOSE AVERAGE THICKNESS IS $\mu = 2.8$ mm

& STANDARD DEVIATION IS $\sigma = .35$ mm.

APPROX. WHAT ~~PROPORTION~~ PERCENT OF SLICES AT THIS PLANT HAVE A THICKNESS BETWEE 1 SD BELOW THE MEAN & 2 S.D. ABOVE THE MEAN?

34% + 34% + 13.5% = 81.5%



WHAT PERCENT OF SLICES ARE EITHER LESS THAN 2.1 mm OF MORE THAN 3.15 mm IN THICKNESS?

100% - 81.5% = 18.5%

CH. 4 : PROBABILITY & PROBABILITY DISTRIBUTIONS

§4.2 EVENTS & THE SAMPLE SPACE

Def: AN EXPERIMENT IS THE PROCESS BY WHICH AN OBSERVATION (MEASUREMENT) IS OBTAINED.

e.g. FLIPPING A COIN (OBSERVE: H OR T)

ROLLING A DIE (OBSERVE: 1, 2, 3, 4, 5, OR 6)

RECORD RAINFALL FOR THE DAY

Def: A SIMPLE EVENT IS THE OUTCOME OBSERVED ON A SINGLE REPITITION OF THE EXPERIMENT.

AN EVENT IS A COLLECTION OF SIMPLE EVENTS.

ALL INDIVIDUAL POSSIBLE OUTCOMES.

e.g. EXPERIMENT : ROLLING A DIE

SIMPLE EVENTS : 1 , 2 , 3 , 4 , 5 , 6

EVENT : ·) ROLLING AN EVEN NUMBER

$\underline{E} = \{2, 4, 6\}$

COLLECTIONS / SETS ARE DENOTED WITH CURLY BRACKETS $\{ \quad \}$

WE SAY THE EVENT E OCCURS IF ANY OF THE SIMPLE EVENTS 2, 4, OR 6 IS OBSERVED.

·) EXPERIMENT : FLIP A COIN 3 TIMES.

SIMPLE EVENTS:

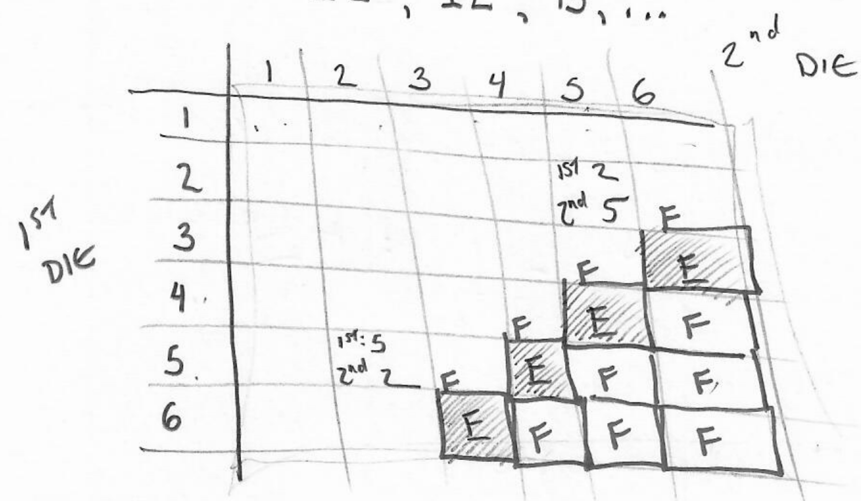
$$\underbrace{\begin{matrix} HHH & HHT & HTH & HTT \\ THH & THT & TTH & TTT \end{matrix}}_{8 \text{ SIMPLE EVENTS}}$$

EVENT E : AT LEAST 2 HEADS (IN 3 FLIPS)

$E = \{HHH, HHT, HTH, THH\}$

ex. EXPERIMENT: ROLL 2 DICE.

SIMPLE EVENTS: 11, 12, 13, ...



36 SIMPLE EVENTS

EVENT: E = ROLL A SUM OF 9

(ADD DICE #'S)

4 SIMPLE EVENTS

= { 6,3 ; 5,4 ; 4,5 ; 3,6 }

EVENT F = ROLL A SUM OF 9 OR MORE

(10 SIMPLE EVENTS)

HW 1.5, 2.2, 2.3, 4.2

LOOK IF YOU HAVE TIME.