HOW MANY WAYS CAN WE ARRANGE / ORDER

4 OBJECTS

ABCD
ABDC
ACBD
ACDB
ADBC
ADCB

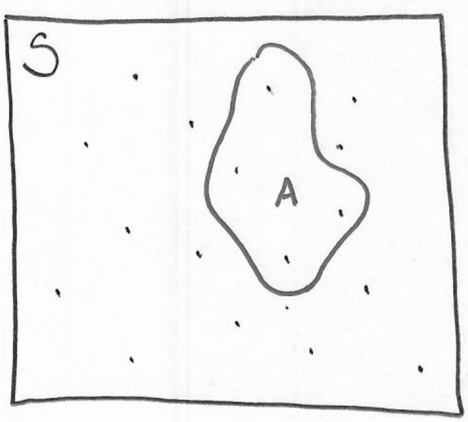
6 WAYS WITH A FIRST

6 WAYS WITH B FIRST

6 C

6 D

<u>SO FAR:</u> TO CALCULATE PROBABILITIES, WE WRITE DOWN ENTIRE SAMPLE SPACE S, WRITE DOWN SIMPLE EVENTS INSIDE OF EVENT OF INTEREST A.



↑ DOTS REPRESENT SIMPLE EVENTS

$$P(A) = \frac{\#\text{ SIMPLE EVENTS INSIDE } A}{\#\text{ SIMPLE EVENTS INSIDE } S}$$

(ASSUMING ALL SIMPLE EVENTS ARE EQUALLY LIKELY.)

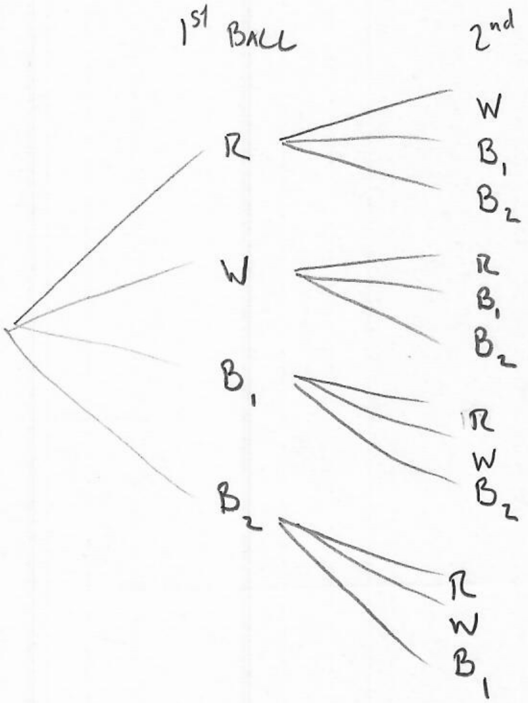
ex. JAR CONTAINS 4 MARBLES: 1 RED
1 WHITE
2 BLUE.

YOU CHOOSE 2 MARBLES AT RANDOM.

(a) FIND PROBABILITY OF SELECTING 2 BLUES.

(b) FIND PROB. OF SELECTING NO RED.

BREAK SAMPLE SPACE DOWN INTO SIMPLE EVENTS, ALL EQUALLY LIKELY.



RW
RB_1
RB_2
WR
$\boxed{WB_1}$
$\boxed{WB_2}$
B_1R
$\boxed{B_1W}$
$\boxed{B_1B_2}$
B_2R
$\boxed{B_2W}$
$\boxed{B_2B_1}$

DEFINE A = 2 BLUE

$$P(A) = \frac{\#(A)}{\#(S)} = \frac{2}{12}$$

NUMBER OF SIMPLE EVENTS

$$= \frac{1}{6} = .1667$$

16.67%

DEFINE E = NO RED MARBLES.

$$P(E) = \frac{\#(E)}{\#(S)} = \frac{6}{12} = \frac{1}{2} = .5$$

$= 50\%$

§ 4.4 USEFUL COUNTING RULES (COMBINATORICS)

SUPPOSE N SIMPLE EVENTS, ALL EQUALLY LIKELY (SAME PROB.). THAT IS, EACH SIMPLE EVENT HAS PROBABILITY $\frac{1}{N}$.

THEN PROBABILITY OF EVENT A

$$P(A) = \frac{n_A}{N}$$

← # OF SIMPLE EVENTS THAT RESULT IN EVENT A.

ALL ABOUT COUNTING!

THE mn RULE

EXPERIMENT IS PERFORMED IN 2 STAGES:

1^{st} STAGE HAS m POSSIBLE OUTCOMES,

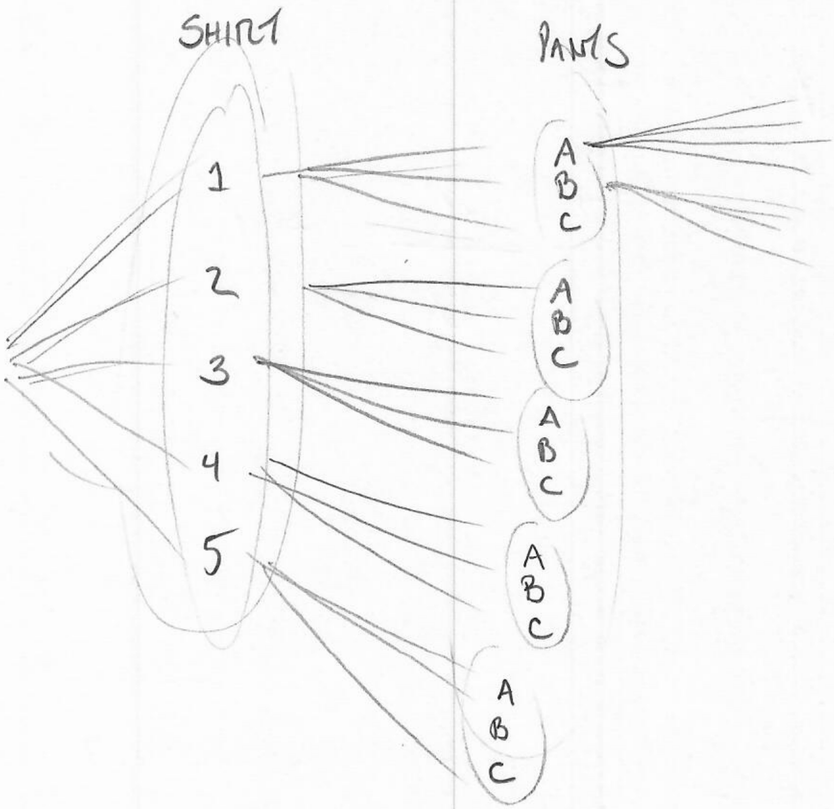
AND FOR EACH OF THESE...

2^{nd} STAGE HAS n POSSIBLE OUTCOMES.

THEN THE EXPERIMENT HAS mn POSSIBLE OUTCOMES.

ex. SUPPOSE AN OUTFIT IS DEFINED AS 1 SHIRT & 1 PAIR OF PANTS. IF I OWN 5 SHIRTS & 3 PAIRS OF PANTS. HOW MANY OUTFITS DO I OWN?

EXPERIMENT: $\underbrace{\text{MAKE AN OUTFIT.}}_{\text{2 STAGES:}}$



$\underbrace{\text{5 GROUPS OF 3}}$

$5 \times 3 = 15$ OUTFITS

EXTENDED mn RULE

EXPERIMENT IS PERFORMED IN k STAGES.

1^{st} STAGE HAS	n_1	POSSIBLE OUTCOMES.
2^{nd}	n_2	
$\vdots$		
k^{TH}	n_k	POSSIBLE OUTCOMES

THE EXPERIMENT HAS

MULT.

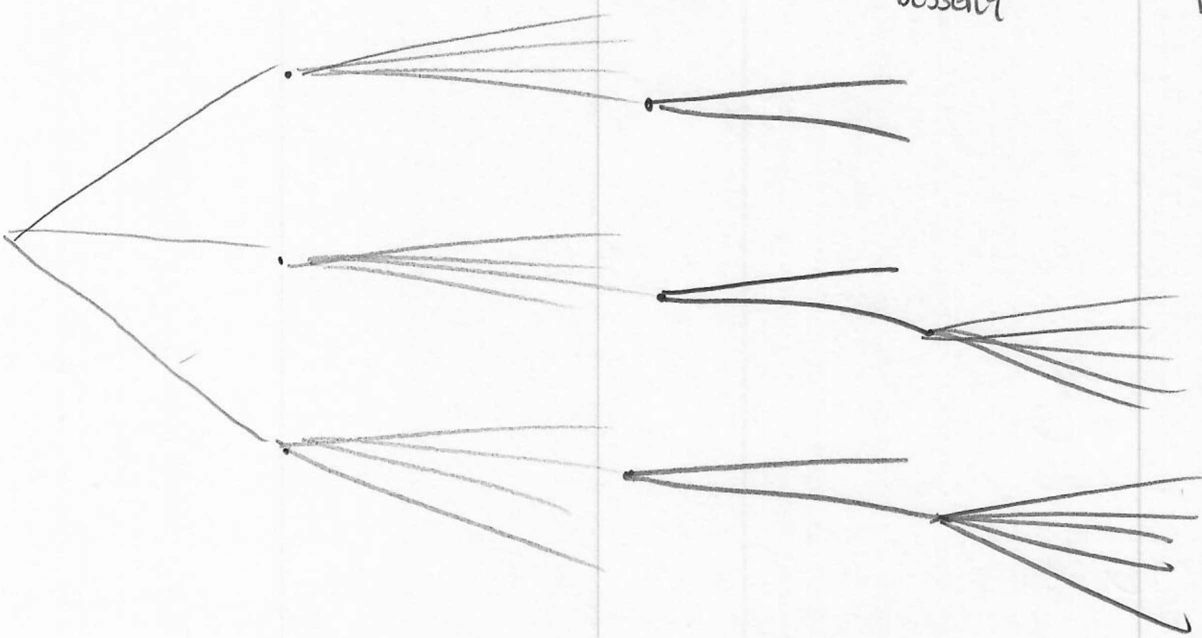
$n_1 n_2 \cdots n_k$

POSSIBLE OUTCOMES.

ex. A RESTAURANT SERVES 3 APP, 4 ENTREES, 2 DESSERTS, 5 DRINKS.

IF A MEAL CONSISTS OF 1 APP, 1 ENTREE, 1 DESSERT, 1 DRINK, HOW MANY MEALS DOES THIS RESTAURANT OFFER?

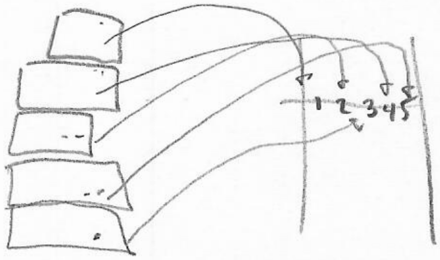
1ST STAGE	2ND STAGE	3RD STAGE	4TH STAGE
APP	ENTREE	DESSERT	DRINK



3	4	2	5

3 GROUPS OF 4 GROUPS OF 2 GROUPS OF 5

$= 3 \times 4 \times 2 \times 5 = 120$ POSSIBLE MEALS ex. SUPPOSE YOU WANT TO ARRANGE 5 BOOKS ON A SHELF. (NEXT TO EACH OTHER). HOW MANY WAYS CAN YOU DO THIS?

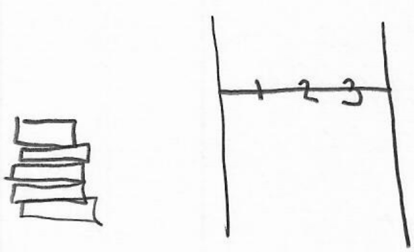


1 WAY.

5 STAGE EVENT: $\underset{1^{st}\text{ BOOK}}{\underline{5^{\text{CHOICES}}}}$ $\underset{2^{nd}\text{ BOOK}}{\underline{4^{\text{CHOICES}}}}$ $\underline{3}$ $\underline{2}$ $\underset{5^{TH}\text{ BOOK}}{\underline{1}}$

GENERAL mn RULE $\Rightarrow$ $5 \times 4 \times 3 \times 2 \times 1 = 120$ POSSIBLE ARRANGEMENTS

ex. 5 BOOKS. HOW MAN WAYS ARE THERE TO SELECT AND ARRANGE 3 OF THE 5 BOOKS?



$$\underbrace{\underset{1^{st}}{\underline{5}} \quad \underset{2^{nd}}{\underline{4}} \quad \underset{3^{rd}}{\underline{3}}}$$

$5 \cdot 4 \cdot 3 = 60$ WAYS

PERMUTATIONS

AN ARRANGEMENT (A PARTICULAR ORDER) IS CALLED A PERMUTATION.

GIVEN n DISTINCT OBJECTS, THE NUMBER OF WAYS TO CHOOSE & ARRANGE (ORDER) r OBJECTS IS GIVEN BY

$$P^n_r = \frac{n!}{(n-r)!}$$

$$\left({}_nP_r \right)$$

WHERE FACTORIAL OPERATOR $!$ IS DEFINED BY

$$a! = a(a-1)(a-2)\cdots(2)(1)$$

(a POSITIVE WHOLE #)

ex. 5 BOOKS. CHOOSE & ARRANGE 3 OF THEM.

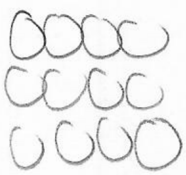
$(5 \times 4 \times 3 = 60$ WAYS$)$ ← mn - RULE

$$P^5_3 = {}_5P_3 = \frac{5!}{(5-3)!} = \frac{5!}{2!}$$

$$= \frac{5 \cdot 4 \cdot 3 \cdot \cancel{2} \cdot \cancel{1}}{\cancel{2} \cdot \cancel{1}} = 60 \text{ WAYS}$$

CLUB

ex. HOW MANY WAYS CAN GROUP OF 12 PEOPLE CHOOSE A PRESIDENT, VICE-PRESIDENT, TREASURER?



B	C	A
A	B	C
P	VP	T

ORDER MATTERS!

PERMUTATION!

HOW MANY WAYS CAN THEY DO THIS?

$$P^{12}_{3} = {}_{12}P_{3} = \frac{12!}{(12-3)!} = \frac{12!}{9!}$$

$$= \frac{12 \cdot 11 \cdot 10 \cdot \cancel{9 \cdots 1}}{\cancel{9 \cdots 1}}$$

$$= 12 \cdot 11 \cdot 10 = \boxed{1320}$$

Q: EXPERIMENT: SELECT 5 CARDS FROM A STANDARD DECK.

HOW MANY WAYS ARE THERE TO DO THIS?

(ASSUME ORDER MATTERS)

$${}_{52}P_{5} = \frac{52!}{(52-5)!} = \frac{52!}{47!}$$

$$= \boxed{311,875,200}$$

1: ~~♠~~ A♥ 5♦ 8♦ J♣ Q♣

2: 5♦ A♥ 8♦ Q♣ J♣

DIFFERENT.

PERMUTATION

COMBINATIONS

GIVEN n DISTINCT OBJECTS, THE NUMBER OF WAYS TO CHOOSE r OBJECTS (ORDER DOESN'T MATTER) IS GIVEN BY

$$C^n_r = {}_nC_r = \frac{n!}{(n-r)!\,r!} \quad \left(= \frac{P^n_r}{r!} \right)$$

3 OBJECTS, CHOOSE 2 (

ABC

SAME: AB, BA

SAME: AC, CA

SAME: BC, CB

$$\left.\begin{array}{l} AB \\ AC \\ BA \\ BC \\ CA \\ CB \end{array}\right\}$$

ORDER MATTERS

$$P^3_2 = \frac{3!}{(3-2)!} = \frac{3!}{1!} = 6$$

ORDER DOESN'T MATTER

DIVIDE BY 2

$$\frac{6}{2} = 3$$

WAYS TO CHOOSE 2 OBJECTS FROM 3 (ORDER DOESN'T MATTER)

NOTE: # WAYS TO ARRANGE n OBJECTS IS $P^n_n = \frac{n!}{(n-n)!} = \frac{n!}{0!} = n!$

$(0! = 1)$

ex.

HOW MANY POKER HANDS ARE THERE?

HOW MANY WAYS ARE THERE TO CHOOSE 5 CARDS FROM A STANDARD DECK, WHEN ORDER DOESN'T MATTER?

52 OBJECTS. CHOOSE 5.

COMBINATION $\longrightarrow$ $_{52}C_5 = \dfrac{52!}{(52-5)!\,5!}$

$$= \frac{52!}{(47!\ 5!)} = \boxed{2,598,960}$$

ex.

8 PERSON COMMITTEE MUST CHOOSE 3 PERSON SUBCOMMITTEE. HOW MANY WAYS CAN THEY DO THIS?

8 OBJECTS, CHOOSE 3.

$$_{8}C_{3} = \frac{8!}{3!\,(8-3)!} = \boxed{56 \text{ WAYS}}$$

ex. YOU HAVE 8 PUPPIES, 5 KITTENS.

HOW MANY WAYS CAN YOU CHOOSE 3 PUPPIES & 2 KITTENS.

2 STAGE EXPERIMENT:

1^{st} PICK PUPPIES $\qquad$ 2^{nd} PICK KITTENS

$${}_8C_3 \times {}_5C_2$$

IN TOTAL $\quad {}_8C_3 \times {}_5C_2 = 56 \times 10 = \boxed{560}$

$$\left(\frac{8!}{3!\,5!} \times \frac{5!}{2!\,3!} = \right)$$

ex. 8 PUPPIES, 5 KITTENS.

YOU CHOOSE 5 ANIMALS AT RANDOM.

FIND PROB. YOU PICK 3 PUPPIES, 2 KITTENS.

LET S = SAMPLE SPACE = $\{P_1 K_2 P_5 K_3 P_2, \ldots\}$

A = EVENT: 3 PUPPIES, 2 KITTENS

$$P(A) = \frac{\#A}{\#S} = \frac{560}{{}_{13}C_5} = \frac{560}{1287} = .4351$$

(43.51%)

ex. SUPPOSE A DRAWER CONTAINS 16 BATTERIES:

13 ARE GOOD, 3 ARE BAD.

IF YOU SELECT 4 BATTERIES AT RANDOM,

FIND THE PROB THAT EXACTLY 2 BATTERIES ARE GOOD.

EXPERIMENT: SELECT 4 BATTERIES FROM 16 BATTERIES.

WAYS TO DO THIS: $\cancel{{}_{16}P_4}$ $\quad {}_{16}C_4$

(# POSSIBLE OUTCOMES OF EXPERIMENT = # S)

~~PERMUTATION ORDER MATTERS~~ COMBINATION ORDER DOESN'T MATTER

$= {}_{16}C_4 = 1820$

EVENT A: SELECTING 2 GOOD BATTERIES OUT OF 13, & 2 BAD BATTERIES OUT OF 3.

2 STAGES: 1st GOOD, 2nd BAD

$$\underbrace{{}_{13}C_2 \times {}_3C_2}_{78 \times 3 = 234}$$

$$P(A) = \frac{\# A}{\# S} = \frac{234}{1820} = .1286 = 12.86\%$$

WHEN ALL SIMPLE EVENTS ARE EQUALLY LIKELY,

$$P(A) = \frac{\# A}{\# S}$$

$\leftarrow$ NUMBER OF SIMPLE EVENTS THAT RESULT IN EVENT A

$\leftarrow$ NUMBER OF SIMPLE EVENTS.

(S = SAMPLE SPACE)

COUNTING

• EXPERIMENT IN k STAGES :

1^{st} STAGE	HAS	n_1 POSSIBLE OUTCOMES
2^{nd} STAGE		n_2
$\vdots$		
k^{th}		n_k

$\Rightarrow$ TOTAL OF $n_1 n_2 \cdots n_k$ POSSIBLE OUTCOMES

• PERMUTATIONS

SELECT & ARRANGE r OBJECTS FROM n OBJECTS

$${}_nP_r = \frac{n!}{(n-r)!}$$

• COMBINATIONS

SELECT r OBJECTS FROM n OBJECTS

$${}_nC_r = \frac{n!}{(n-r)!\,r!}$$