4.32 b. HOW MANY WAY ARE THERE TO GET 4 OF A KIND IN POKER.

5 CARDS : 4 HAVE SAME DENOMINATION (e.g. 4 SEVENS, 4 JACKS, ETC.)

WRITE DOWN 2-3 SPECIFIC 4 OF A KINDS (5 CARD HAND)

4♡ 4♢ 4♧ 4♤ | J♢

K♢ K♡ K♤ K♧ | A♤

① CHOOSE DENOMINATION ("KIND") ${}_{13}C_1$

② CHOOSE 5TH CARD ${}_{48}C_1$

→ TOTAL : 2-STAGE EVENT

$${}_{13}C_1 \cdot {}_{48}C_1 = 13 \cdot 48 = \boxed{624}$$

c. PROBABILITY OF BEING DEALT 4-OF-A-KIND :

TOTAL # OF 5-CARD HANDS : ${}_{52}C_5 = 2{,}598{,}960$

TOTAL # OF 4-OF-A-KINDS : 624

$$P(\text{4-of-a-kind}) = \frac{624}{2{,}598{,}960} = .00024009\ldots$$

CHOOSE DENOMINATION, CHOOSE 4 CARDS, CHOOSE 5TH CARD

$_{13}C_1$, $\underbrace{_4C_4}_{1} = \frac{4!}{4!\,0!} = 1$, $_{48}C_1$.

4.39

EXPERIMENT: ARRANGE 12 OBJECTS:

3 Δ's
3 □'s
3 ▭'s
3 O's

TOTAL # OF WAYS TO DO THIS: $_{12}P_{12} = \frac{12!}{0!} = 12! =$

$= 479{,}001{,}600$

EVENT: ARRANGE THE BLOCKS BY SHAPE e.g. $\Delta\Delta\Delta$ | OOO | □□□ | ▭▭▭

OOO ▭▭▭ $\Delta\Delta\Delta$ □□□

5 STAGE EVENT

1 CHOOSE ARRANGEMENT FOR SHAPES: $_4P_4 = 4!$

2 ARRANGE Δ's $_3P_3 = 3!$

3 ARRANGE □'s $_3P_3 = 3!$

4 ARRANGE ▭ $_3P_3 = 3!$

5 ARRANGE O's $_3P_3 = 3!$

$\Delta_1 \Delta_2 \Delta_3$

$\Delta_3 \Delta_1 \Delta_2$

WAYS TO ARRANGE BLOCKS BY SHAPE: $4!\,3!\,3!\,3!\,3!$

$(24)(6)(6)(6)(6) = 31{,}104$

$$P(\text{RANDOMLY ARRANGING BLOCKS BY SHAPE}) = \frac{31{,}104}{479{,}001{,}600}$$

$$\approx .0000649 \text{ or } .00649\%$$

△△△

○□▭ △△△ ○□□○▭ ...

ex. FARM HAS 5 GOATS
7 SHEEP
9 PIGS

YOU PICK 5 ANIMALS AT RANDOM.

FIND PROB. $\boxed{\text{YOU PICK 2 GOATS, 2 SHEEP, 1 PIG.}}$

A = PICK 2G, 2S, 1P

$\#A = \underline{\text{CHOOSE GOATS}}, \underline{\text{CHOOSE SHEEP}}, \underline{\text{CHOOSE PIG}}$

$$_5C_2 \cdot {_7C_2} \cdot {_9C_1}$$

$$= 10 \cdot 21 \cdot 9$$

$$= \boxed{1890}$$

EXPERIMENT: CHOOSE 5 ANIMALS FROM 21 ANIMALS.

SAMPLE SPACE S = { ALL COMBINATIONS OF 5 ANIMALS CHOSEN FROM 21 ANIMALS }

$$\#S = {_{21}C_5} = 20{,}349$$

$$\frac{21!}{5!(21-5)!} = \frac{21!}{5!\,16!} = \frac{21 \cdot 20 \cdot 19 \cdot 18 \cdot 17}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$$

$$P(A) = \frac{\#A}{\#S} = \frac{1890}{20349} = .0929$$

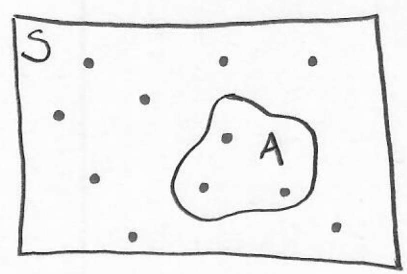
§4.5 EVENT RELATIONS & PROBABILITY RULES

SET THEORY

RECALL: SAMPLE SPACE S IS THE SET (COLLECTION) OF ALL POSSIBLE OUTCOMES OF AN EXPERIMENT.

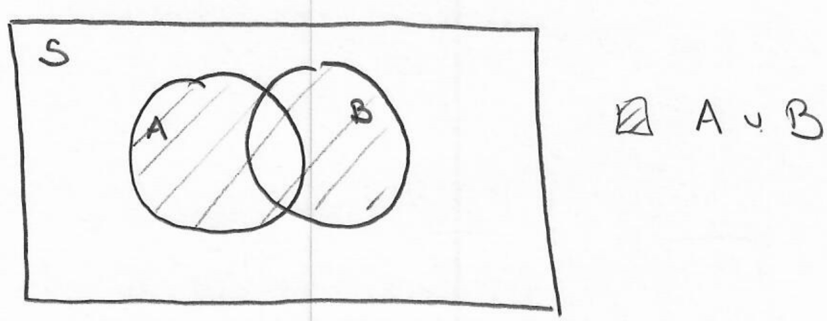
AN EVENT IS A SUBSET OF S.
(A COLLECTION OF SOME POSSIBLE OUTCOMES)

VENN DIAGRAM:



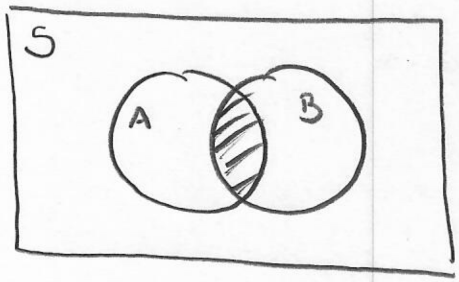
Def: THE UNION OF TWO EVENTS, A AND B, IS THE EVENT THAT EITHER A OCCURS, B OCCURS, OR BOTH A AND B OCCUR.

NOTATION $A \cup B$ "A UNION B"



Def: THE INTERSECTION OF TWO EVENTS, A AND B, IS THE EVENT THAT BOTH A AND B OCCUR. NOTATION $A \cap B$

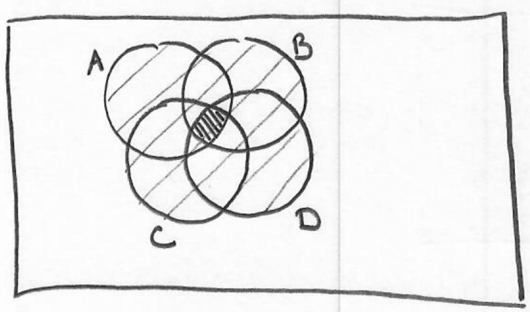
"A INTERSECT B"



$A \cap B$

"OVERLAP"

ex.

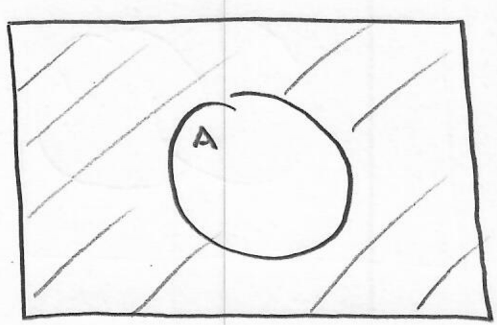


$A \cup B \cup C \cup D$

$A \cap B \cap C \cap D$

Def: THE COMPLEMENT OF AN EVENT A IS THE EVENT THAT A DOES NOT OCCUR (NOT A).

NOTATION A^C (A')



A^C

ex.

EXP: TWO FAIR COINS ARE TOSSED.

EVENTS: A = AT LEAST ONE HEAD

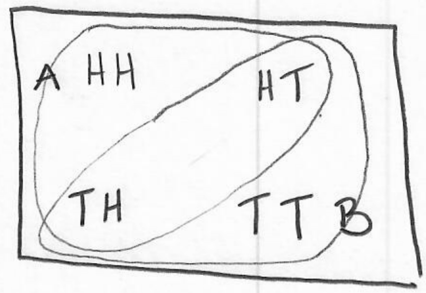
B = AT LEAST ONE TAIL

$$A \cup B = \{HH, HT, TH, TT\}$$

$$A \cap B = \{HT, TH\}$$

$$A^c = \{TT\}$$

$$B^c = \{HH\}$$



ex.

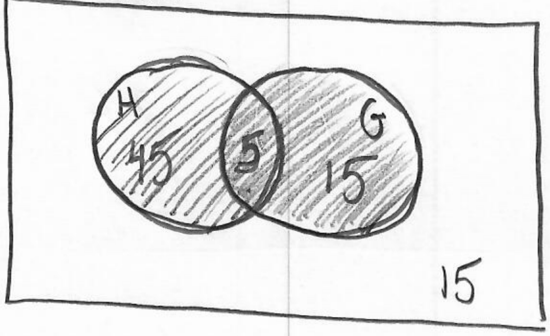
80 CHILDREN ON A PLAY GROUND.

50 WEAR A HAT

20 WEAR GLOVES

15 DO NOT WEAR A HAT & DO NOT WEAR GLOVES.

80 CHILDREN



$$\#(H \cup G) = 65$$

$$\#H + \#G = \#(H \cup G) + \#(H \cap G)$$

$$\#H + \#G - \#(H \cap G) = \#(H \cup G)$$

$$50 + 20 - \underbrace{\#(H \cap G)}_{=} = 65$$

GIVE 2 SETS A, B:

$$\frac{\#A}{\#S} + \frac{\#B}{\#S} - \frac{\#(A \cap B)}{\#S} = \frac{\#(A \cup B)}{\#S}$$

$$\boxed{P(A) + P(B) - P(A \cap B) = P(A \cup B)}$$

ADDITION RULE FOR 2 EVENTS A, B.

ex. SUPPOSE THE PROBABILITY OF ENGINE COMPONENT A FAILING IS 5%, PROBABILITY OF ENGINE COMPONENT B FAILING IS 10%. SUPPOSE PROB. OF BOTH COMPONENTS FAILING IS 2%. FIND PROBABILITY THAT AT LEAST ONE OF THESE COMPONENTS FAILS.

$$\underbrace{\text{A FAILS, B FAILS, OR BOTH A \& B FAIL.}}_{A \cup B}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= 5\% + 10\% - 2\% = \boxed{13\%}$$

Note: 2 EVENTS A, B ARE MUTUALLY EXCLUSIVE IF THEY CANNOT BOTH OCCUR AT THE SAME TIME.

i.e. $P(A \cap B) = 0$ ← DEF. MUTUALLY EXCLUSIVE.

- A & A^c ARE MUTUALLY EXCLUSIVE.

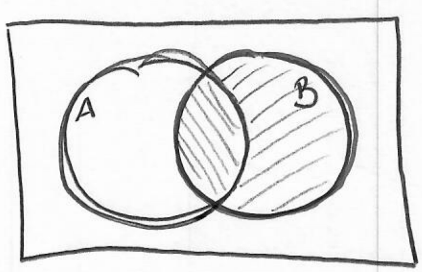
- $A \cup A^c = S$ (ENTIRE SAMPLE SPACE)

$$\Rightarrow \quad \underbrace{P(A \cup A^c)} = P(A) + P(A^c) - \underbrace{P(A \cap A^c)}_{0}$$

$$P(S) = \boxed{1 = P(A) + P(A^c)}$$

$$P(A^c) = 1 - P(A)$$

COMPLEMENT RULE

- 

$$B = \underbrace{(B \cap A^c) \cup (B \cap A)}$$

MUTUALLY EXCLUSIVE

$$P(B) = P(B \cap A^c) + P(B \cap A) - \underbrace{P((B \cap A^c) \cap (B \cap A))}_{0}$$

$$\boxed{P(B) = P(B \cap A^c) + P(B \cap A)}$$

p. 143 EXAMPLE 4.5

	A	B	C
D	$P(A \cap D)$	$P(B \cap D)$	$P(C \cap D)$
E	$P(A \cap E)$	$P(B \cap E)$	$P(C \cap E)$

MUTUALLY EXCLUSIVE

1. $P(D) = P\big((D \cap A) \cup (D \cap B) \cup (D \cap C)\big)$

$= P(D \cap \underline{A}) + P(D \cap \underline{B}) + P(D \cap \underline{C})$

$= .35 + .08 + .01 = .44$

2. $P(E) = P(D^c) = 1 - P(D) = 1 - .44 = .56$

(P(NO CHILD IN COLLEGE) = P((CHILD IN COLLEGE)^c))

3. .69

ADDITION RULE

$P(A \cup D) = P(A) + P(D) - P(A \cap D)$

TOO HIGH CHILD

$= .6 + .44 - .35$

$= .69$