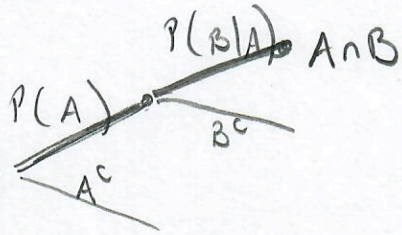
EXAM # 1 NEXT TUE. 6/23 4.7 TODAY

SECTIONS $\boxed{1.5,\ 2.2-4,\ 4.2-8,\ 5.2}$

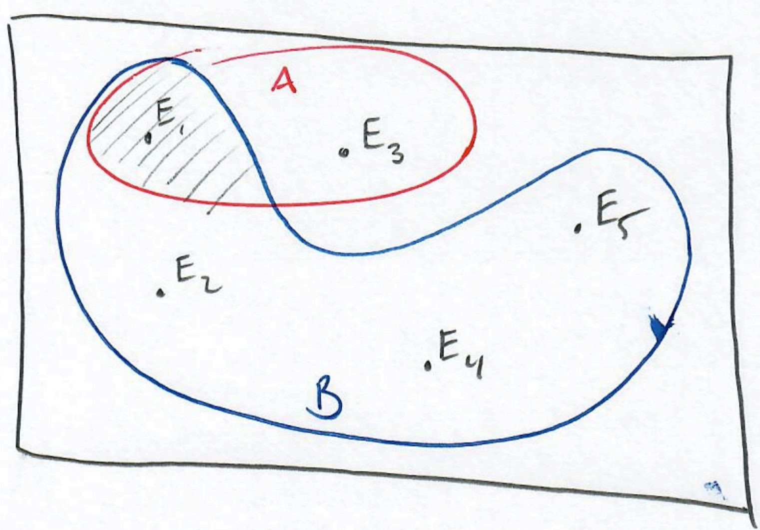
4.40 (b). $P(A \cap B) = P(A)P(B|A)$

$= P(B)P(A|B)$



IND: $P(A|B) = P(A)$

$P(B|A) = P(B)$



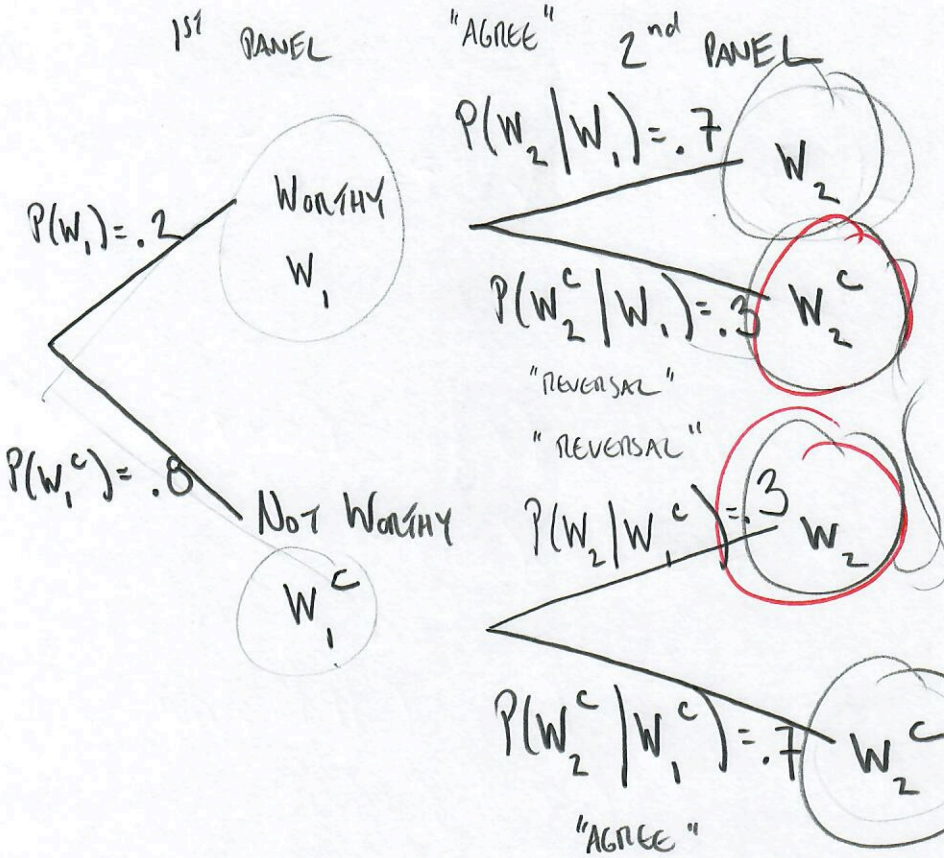
$A \cap B = \{E_1\}$

$P(A \cap B) = P(E_1) = \boxed{\frac{1}{5}}$ SAME

$$\begin{aligned} P(A \cap B) &= P(B)P(A|B) \\ &= (.8)(.25) \\ &= \boxed{.2} \end{aligned}$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{1/5}{4/5} = \frac{1}{4} = .25$$

4.55



$P(W_1)P(W_2|W_1)$

$P(W_1 \cap W_2) = (.2)(.7)$

$= .14$ a.

$.06$

$.24$

$\left.\begin{matrix} .06 \\ .24 \end{matrix}\right\} .3$

$P(W_1^c \cap W_2^c)$

$= P(W_1^c)P(W_2^c|W_1^c)$

$= (.8)(.7) = .56$

b.

MUTUALLY EXCLUSIVE

$P(\text{APPROVED BY 1 PANEL})$

$= P((W_1 \cap W_2^c) \cup (W_1^c \cap W_2))$

$= P(W_1 \cap W_2^c) + P(W_1^c \cap W_2)) - \overbrace{P((W_1 \cap W_2^c) \cap (W_1^c \cap W_2))}^{0}$

$= P(W_1)P(W_2^c|W_1) + P(W_1^c)P(W_2|W_1^c)$

$= (.2)(.3) + (.8)(.3) = \boxed{.3}$

EXAM
PLAN : A. WEBWORK (MAA)

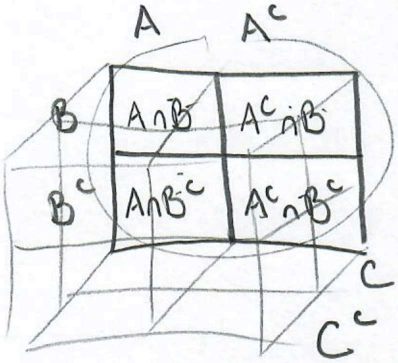
ONLINE MATH PLATFORM (FREE)

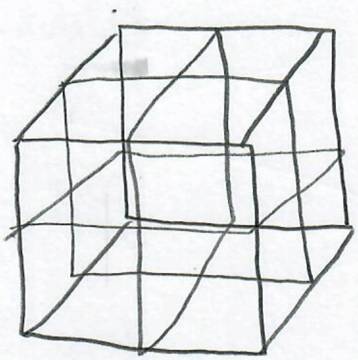
B. BLACKBOARD

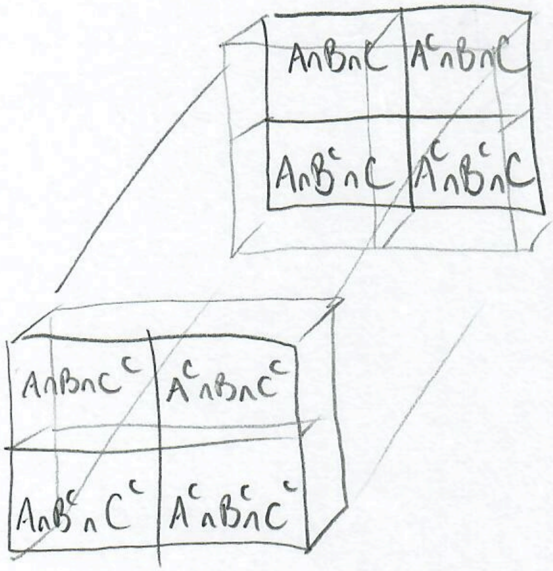
EMAIL THIS WEEK WITH INSTRUCTIONS

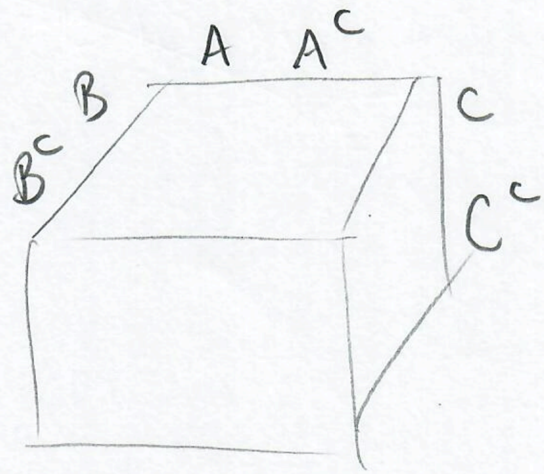
EXAM ONLINE 10:30 - 12:10 PM (TIMED)

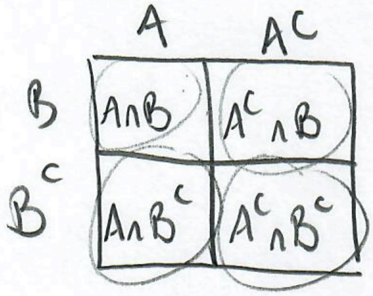
TUES. 6/23.

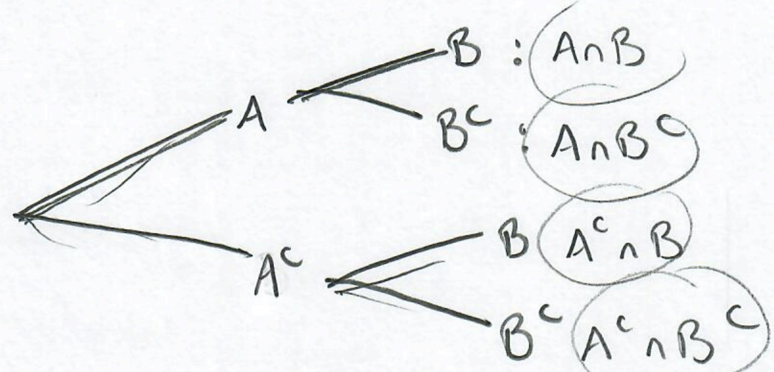

A
A^c
B
B^c
$A \cap B$
$A^c \cap B$
$A \cap B^c$
$A^c \cap B^c$
C
C^c




$A \cap B \cap C$
$A^c \cap B \cap C$
$A \cap B^c \cap C$
$A^c \cap B^c \cap C$
$A \cap B \cap C^c$
$A^c \cap B \cap C^c$
$A \cap B^c \cap C^c$
$A^c \cap B^c \cap C^c$


A
A^c
B
B^c
C
C^c


A
A^c
B
B^c
$A \cap B$
$A^c \cap B$
$A \cap B^c$
$A^c \cap B^c$


A
A^c
B : $A \cap B$
B^c : $A \cap B^c$
B $A^c \cap B$
B^c $A^c \cap B^c$

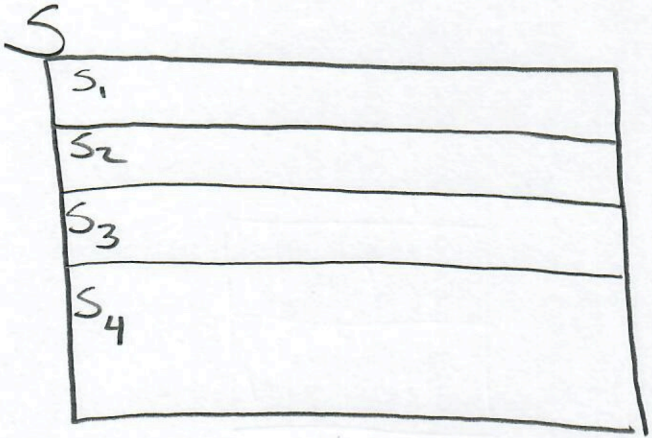
§4.7 BAYES' RULE

1. LAW OF TOTAL PROBABILITY
2. BAYES' RULE

SUPPOSE SAMPLE SPACE CAN BE BROKEN UP INTO k SUBSPACES: $S_1, S_2, \dots, S_k$. SUPPOSE THE k SUBSPACES ARE

(1) MUTUALLY EXCLUSIVE

(2) EXHAUSTIVE (TOGETHER, THEIR UNION, IS THE ENTIRE SAMPLE SPACE)



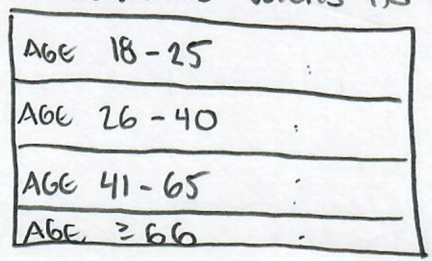
4 SUBSPACES

(1) $S_i \cap S_j = \emptyset$
$(i \neq j)$

(2) $S_1 \cup S_2 \cup \dots \cup S_k = S$

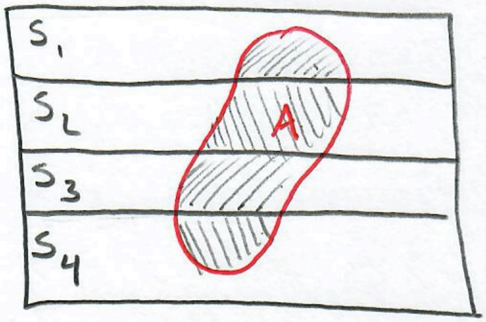
e.g.

ALL REGISTERED VOTERS IN U.S.



"DEMOGRAPHICS"

e.g. STATE,
EMPLOYMENT STATUS,



MUTUALLY EXCLUSIVE

$$P(A) = P\big((A \cap S_1) \cup (A \cap S_2) \cup (A \cap S_3) \cup (A \cap S_4) \big)$$

$$= P(A \cap S_1) + P(A \cap S_2) + P(A \cap S_3) + P(A \cap S_4)$$

LAW OF TOTAL PROB: IF $\boxed{S = S_1 \cup S_2 \cup \dots \cup S_K ,}$

THE S_i'S ARE ALL MUTUALLY EXCLUSIVE THEN

FOR ANY EVENT A, $P(A) = P(A \cap S_1) + P(A \cap S_2) + \dots + P(A \cap S_K)$

$\left(\text{RECALL } P(A \cap B) = P(B)\,P(A|B) \right)$

$$\therefore \boxed{P(A) = P(S_1)P(A|S_1) + P(S_2)P(A|S_2) + \dots P(S_K)P(A|S_K)}$$

ex.

EXPERIMENT : SELECT 1 RESIDENT OF NYC AT RANDOM.

SAMPLE SPACE = POPULATION OF NYC

("SUBSPACES" = "SUBPOPULATIONS")

5 SUBSPACES
(5 SUBPOPULATIONS)

19% OF NYC RESIDENTS LIVE IN MANHATTAN (2015)

31% OF BROOKLYN

27% QUEENS

17% BRONX

6% STATEN ISLAND

29% OF MANHATTAN RESIDENTS ARE FOREIGN BORN

38% BK

49% Q

32% BX

21% SI

	S_1 M	S_2 BK	S_3 Q	S_4 BX	S_5 SI
PROPORTION OF NYC	.19	.31	.27	.17	.06
PROPORTION FOREIGN BORN	.29	.38	.49	.32	.21

$P(S_4) = .17$

LET F = FOREIGN BORN

$P(F|S_4) = .32$

WHAT IS THE PROBABILITY THAT A NYC RESIDENT IS FOREIGN BORN?

(INTUITION: BETWEEN .21 & .49)

LAW OF TOTAL PROBABILITY:

$$P(F) = P(F \cap S_1) + P(F \cap S_2) + P(F \cap S_3) + P(F \cap S_4) + P(F \cap S_5)$$

$$= P(S_1)P(F|S_1) + P(S_2)P(F|S_2) + P(S_3)P(F|S_3) + P(S_4)P(F \cap S_4) + P(S_5)P(F \cap S_5)$$

* ADD UP TO 1

$$= \underbrace{(.19)(.29) + (.31)(.38) + (.27)(.49) + (.17)(.32) + (.06)(.21)}_{\text{WEIGHTED AVERAGE OF } P(F \cap S_i)}$$

$$= .3722$$

BAYES' RULE:

RECALL: DEF OF CONDITIONAL PROBABILITY

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$P(B)\,P(A|B) = P(A \cap B)$$ GENERAL MULT. RULE

$$\therefore \quad P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)\,P(B|A)}{P(B)}$$

$$\boxed{P(A|B) = \frac{P(A)\,P(B|A)}{P(B)}}$$

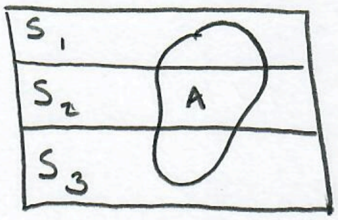
GIVES ONE CONDITIONAL PROBABILITY

IN TERMS OF ITS OPPOSITE

BAYE'S RULE +

LAW OF

BAYES' RULE + LAW OF TOTAL PROB.



$$P(S_i|A) = \frac{P(S_i)P(A|S_i)}{P(A)} \quad \text{(BAYES' RULE)}$$

L.O.T.P.

$$\boxed{P(S_i|A) = \frac{P(S_i)P(A|S_i)}{P(S_1)P(A|S_1) + P(S_2)P(A|S_2) + \ldots + P(S_k)P(A|S_k)}}$$

ex. THERE IS A TEST FOR A DISEASE :

99.9% OF PEOPLE WITH THE DISEASE TEST POSITIVE.

98% OF PEOPLE WITHOUT THE DISEASE TEST NEGATIVE.

SUPPOSE .5% OF POPULATION HAS THE DISEASE.

WHAT IS THE PROB. THAT A PERSON WHO TESTS POSITIVE HAS THE DISEASE ?

NOTATION

LET : S_1 = SUBPOPULATION THAT HAS THE DISEASE

S_2 = SUBPOPULATION THAT DOES NOT HAVE THE DISEASE

A = TEST POSITIVE FOR THE DISEASE

A^c = TEST NEGATIVE FOR THE DISEASE