§4.7 BAYES' RULE (& LAW OF TOTAL PROBABILITY) CONTINUED

ex. SUPPOSE ON A PARTICULAR DAY,

MUTUALLY EXC.
EXHAUSTIVE

70% OF STUDENTS TAKE SUBWAY TO SCHOOL
15% DRIVE TO SCHOOL
10% BIKE TO SCHOOL
5% CAB TO SCHOOL

COND. PROB.

20% OF STUDENTS WHO TOOK SUBWAY WERE LATE TO SCHOOL
12% " DROVE " "
5% " BIKED "
30% " CAB "

WHAT'S THE PROBABILITY THAT A STUDENT IS LATE TO SCHOOL?

IF A STUDENT IS LATE, WHAT IS THE PROB THAT THEY TOOK THE SUBWAY TO SCHOOL?

① NOTATION: SUBPOPULATIONS

S_1 = SUBWAY
S_2 = DRIVES
S_3 = BIKE
S_4 = CAB

A = LATE TO SCHOOL.

② SUMMARIZE GIVEN INFO WITH NOTATION:

$P(S_1) = .70$	$P(A \mid S_1) = .20$
$P(S_2) = .15$	$P(A \mid S_2) = .12$
$P(S_3) = .10$	$P(A \mid S_3) = .05$
$P(S_4) = .05$	$P(A \mid S_4) = .30$

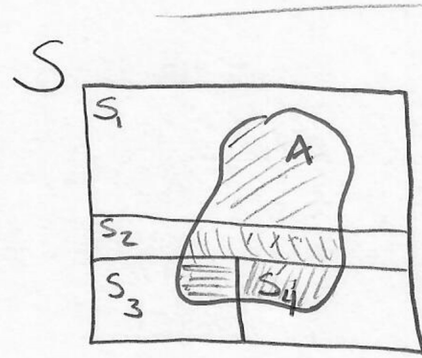
ADD UP TO 1

DO NOT HAVE TO ADD TO 1.

NOT ALL THE SAME. SO A IS DEPENDENT ON S_1, S_2, S_3, S_4 (NOT INDEPENDENT)

③ WHAT DO WE NEED TO FIND?

FIND $P(A)$.

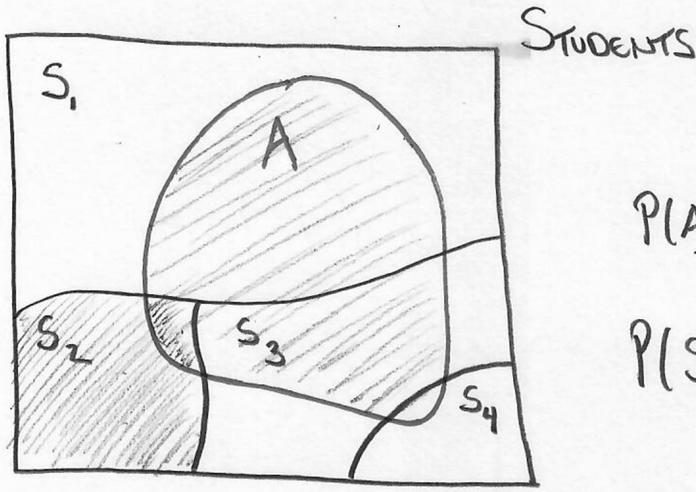


LAW OF TOTAL PROBABILITY:

$$P(A) = P(A \cap S_1) + P(A \cap S_2) + P(A \cap S_3) + P(A \cap S_4)$$

$$\boxed{P(A) = P(S_1)P(A|S_1) + P(S_2)P(A|S_2) + P(S_3)P(A|S_3) + P(S_4)P(A|S_4)}$$

$$P(A) = (.7)(.2) + (.15)(.12) + (.10)(.05) + (.05)(.30)$$



$P(A|S_2)$

$P(S_2|A)$

$$P(S_1) + P(S_2) + P(S_3) + P(S_4) = 1$$

$$P(S_1|A) + P(S_2|A) + P(S_3|A) + P(S_4|A) = 1$$

$P(A) = \boxed{.178}$ ← PROB A STUDENT IS LATE TO SCHOOL.

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③ WHAT IS BEING ASKED?

FIND $P(S_1 | A)$.

(WE'VE BEEN TOLD $P(A|S_1)$. WE WANT TO SWITCH THE CONDITION)

BAYES' RULE!

$$\boxed{P(S_1|A) = \frac{P(S_1)P(A|S_1)}{P(A)}}$$

$$P(S_i|A) = \frac{P(S_i)P(A|S_i)}{P(A)}$$

$$P(S_1|A) = \frac{(.7)(.2)}{.178} = .7865$$

.178 FOUND USING L.O.T.P.

(78.65%)

$$P(S_2|A) = \frac{P(S_2)P(A|S_2)}{P(A)} = \frac{(.15)(.12)}{.178} = .1011$$

(10.11%)

$$P(S_3|A) = \frac{P(S_3)P(A|S_3)}{P(A)} = \frac{(.10)(.05)}{.178} = .0281$$

(2.81%)

$$P(S_4|A) = \frac{P(S_4)P(A|S_4)}{P(A)} = \frac{(.05)(.3)}{.178} = .0843$$

(8.43%)

ex. 4.75

NOTATION: S_1 = PLAY GOES TO LEFT

S_2 = PLAY GOES TO RIGHT

A = RIGHT GUARD TAKES SHIFTED STANCE.

A^c = RIGHT GUARD TAKES BALANCED STANCE.

GIVEN: $P(S_1) = .3$ $\quad P(A^c|S_1) = .9 \quad P(A|S_1) = .1$

$P(S_2) = .7 \quad P(A|S_2) = .8$

$P(A^c|S_2) = .2$

BAYES' RULE

(a) $P(S_1|A^c)$ PROB. PLAY GOES TO LEFT GIVEN GUARD TAKES BALANCED STANCE.

$$P(S_1|A^c) = \frac{P(S_1)P(A^c|S_1)}{P(A^c)} \quad \text{BAYES' RULE}$$

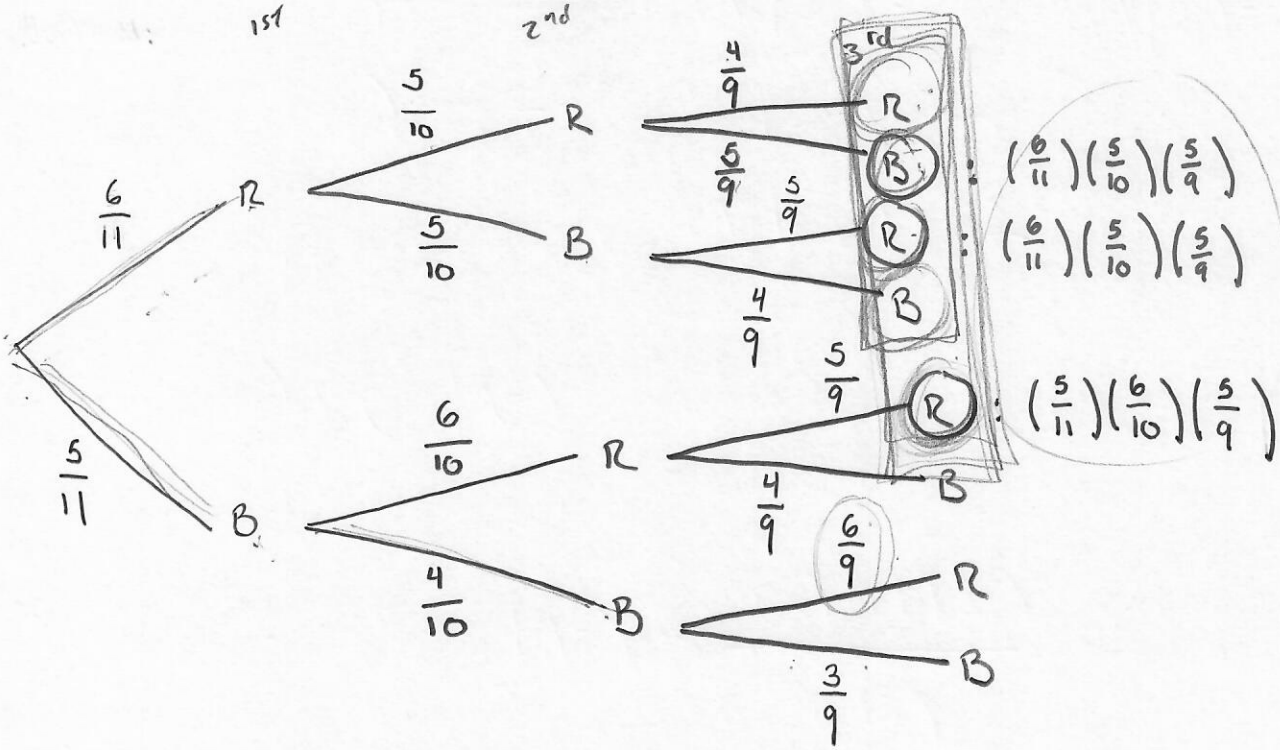
$$\overset{\text{L.O.T.P.}}{=} \frac{P(S_1)P(A^c|S_1)}{P(S_1)P(A^c|S_1) + P(S_2)P(A^c|S_2)}$$

$$= \frac{(.3)(.9)}{(.3)(.9) + (.7)(.2)} = \boxed{.6584}$$

(b) $P(S_2|A^c) = 1 - P(S_1|A^c) = 1 - .6584 = \boxed{.3416}$

ex. ALICE IS TAKING MARBLES OUT OF A JAR 1 BY 1 FROM A JAR WITH 6 RED & 5 BLUE MARBLES. AFTER TAKING OUT 3 MARBLES, SHE HAS 2 RED & 1 BLUE MARBLE.

WHAT IS THE PROB THAT THE FIRST MARBLE WAS RED?

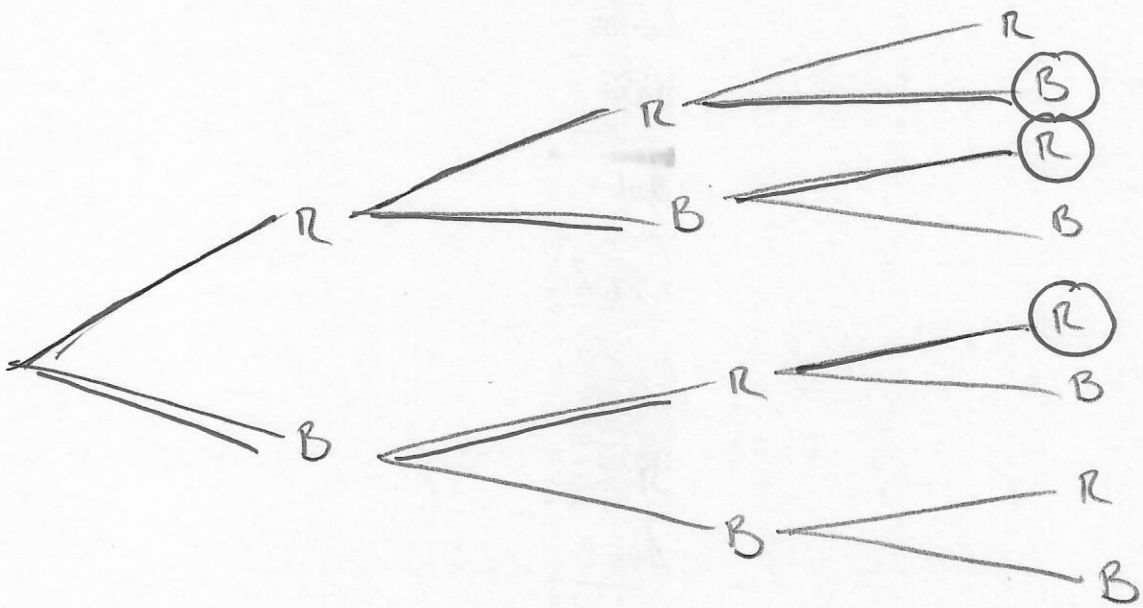


NOTATION: A = ALICE SELECTS 2 RED & 1 BLUE

B = ALICE'S FIRST MARBLE IS RED.

FIND: $P(B|A) \overset{DEF}{=} \dfrac{P(B \cap A)}{P(A)} = \dfrac{2\left(\frac{6}{11}\right)\left(\frac{5}{10}\right)\left(\frac{5}{9}\right)}{2\left(\frac{6}{11}\right)\left(\frac{5}{10}\right)\left(\frac{5}{9}\right) + \left(\frac{5}{11}\right)\left(\frac{6}{10}\right)\left(\frac{5}{9}\right)}$

$= \boxed{.6667}$



SIMPLE EVENTS

$A = $ 2 Red 1 Blue $= \{RRB, RBR, BRR\}$

$$P(A) = P(\{RRB, RBR, BRR\})$$

$$= P(RRB) + P(RBR) + P(BRR)$$

$$P(\geq 1 \text{ RED}) = P((\text{No RED})^c)$$

$$= 1 - P(\text{NO RED})$$

$$= 1 - P(BBB)$$

$$= 1 - \left(\frac{5}{11}\right)\left(\frac{4}{10}\right)\left(\frac{3}{9}\right)$$

$$= \boxed{.9394}$$

4.78

NOTATION: S_1 = CRIME

S_2 = MISTAKE

S_3 = CORRECT RETURN

A = DENY KNOWLEDGE OF ERROR WHEN CONFRONTED BY INVESTIGATOR.

GIVEN:

$P(S_1) = .05$ $\quad$ $P(A|S_1) = .8$

$P(S_2) = .02$ $\quad$ $\underbrace{P(A|S_2) = 1}_{\text{GUARANTEED}}$

$P(S_3) = .93$ $\quad$ $\underbrace{P(A|S_3) = 0}_{\text{WON'T HAPPEN}}$

FIND: $$P(S_1|A) = \frac{P(S_1)P(A|S_1)}{P(A)}$$

$$= \frac{P(S_1)P(A|S_1)}{P(S_1)P(A|S_1) + P(S_2)P(A|S_2) + P(S_3)P(A|S_3)}$$

$$= \frac{(.05)(.8)}{(.05)(.8) + (.02)(1) + (.93)(0)}$$

$$= \quad = .6667 \text{ or } 66.67\%$$