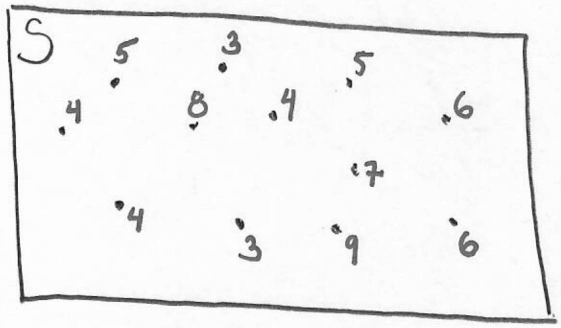
§4.8 DISCRETE RANDOM VARIABLES & THEIR PROBABILITY DISTRIBUTIONS

RANDOM VARIABLES :



SAMPLE SPACE S CONTAINS FINITE NUMBER OF SIMPLE EVENTS

e.g. EXP. PICK A STUDENT AT RANDOM.
RV: NUMBER OF LETTERS IN STUDENT'S NAME.

ASSIGN TO EACH SIMPLE EVENT A NUMBER.

(FINITE # OF SIMPLE EVENTS
$\Rightarrow$ ONLY FINITE OF # OF NUMBERS ARE USED.)

<u>Def</u>: A RANDOM VARIABLE IS A RULE (FUNCTION) THAT ASSIGN A REAL # TO EVERY SIMPLE EVENT.

e.g. EXP. PICK A TIGER AT RANDOM
RV : # STRIPES ON THAT TIGER.

EXP : PICK A RANDOM US CITIZEN
RV: # SHOES THE PERSON OWNS . .
(RV: # FINGERS THE PERSON HAS).

RANDOM VARIABLE $\neq$ RANDOM #
RANDOM VARIABLE = # DETERMINED BY RANDOM EVENT.

DISCRETE RANDOM VARIABLES TAKE ONLY A FINITE # OF VALUES.

ex. EXP: ROLL 2 DICE.

SAMPLE SPACE

EACH SQUARE REPRESENTS A SIMPLE EVENT.

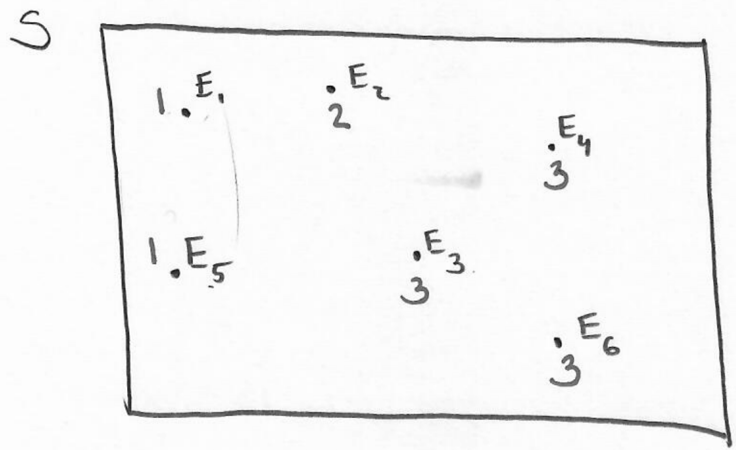
1st DIE

2nd DIE	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

RAND VAR X : ADD THE DICE

POSSIBLE VALUES FOR RANDOM VARIABLE X : 2, 3, 4, ..., 12.

<u>Def</u>: THE <u>PROBABILITY DISTRIBUTION</u> FOR A DISCRETE RANDOM VARIABLE IS A FORMULA, <u>TABLE</u>, GRAPH (RULE) THAT LISTS THE POSSIBLE VALUES FOR X, AND THE PROBABILITY $p(x)$ ASSOCIATED WITH EACH VALUE OF X.



E_i = SIMPLE EVENTS
R.V. ASSOCIATED WITH EACH SIMPLE EVENT.

RANDOM VARIABLE X TAKES ON VALUES : 1, 2, 3

$$p(x=1) = P(\{E_1, E_5\}) = P(E_1) + P(E_5)$$

EVENT

EVENTS THAT CAUSE R.V. TO EQUAL 1.

$$p(x=2) = P(E_2)$$

$$p(x=3) = P(\{E_3, E_4, E_6\})$$

$$= P(E_3) + P(E_4) + P(E_6)$$

ex. PROBABILITY DISTRIBUTION FOR X = SUM OF #'S SHOWN ON 2 DICE

x	2	3	4	5	6	7	8	9	10	11	12
$p(x)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

PROBABILITY THAT SUM = 5 : $p(x=5) = p(5) = \frac{4}{36}$

exp. SUPPOSE YOU BUY 1 RAFFLE TICKET.

5,000 RAFFLE TIX SOLD.

$5000 - 14 = 4986$ LOSING TIX

1 TOP PRIZE OF $\$1{,}000 \to x = 998$

3 2^{nd} PRIZES OF $\$500 \to x = 498$

10 3^{rd} PRIZES OF $\$75 \to x = 73$

ALL THE REST WIN $\$0 \to x = -2$

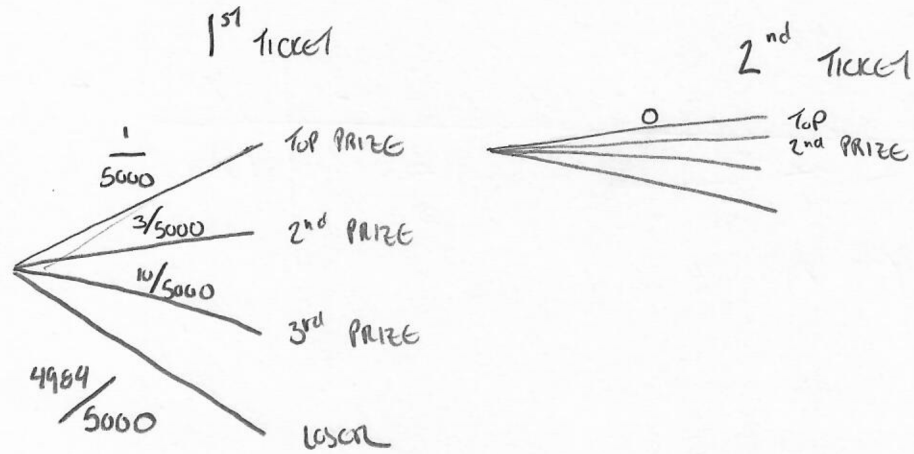
TIX COST $\$2$ EACH.

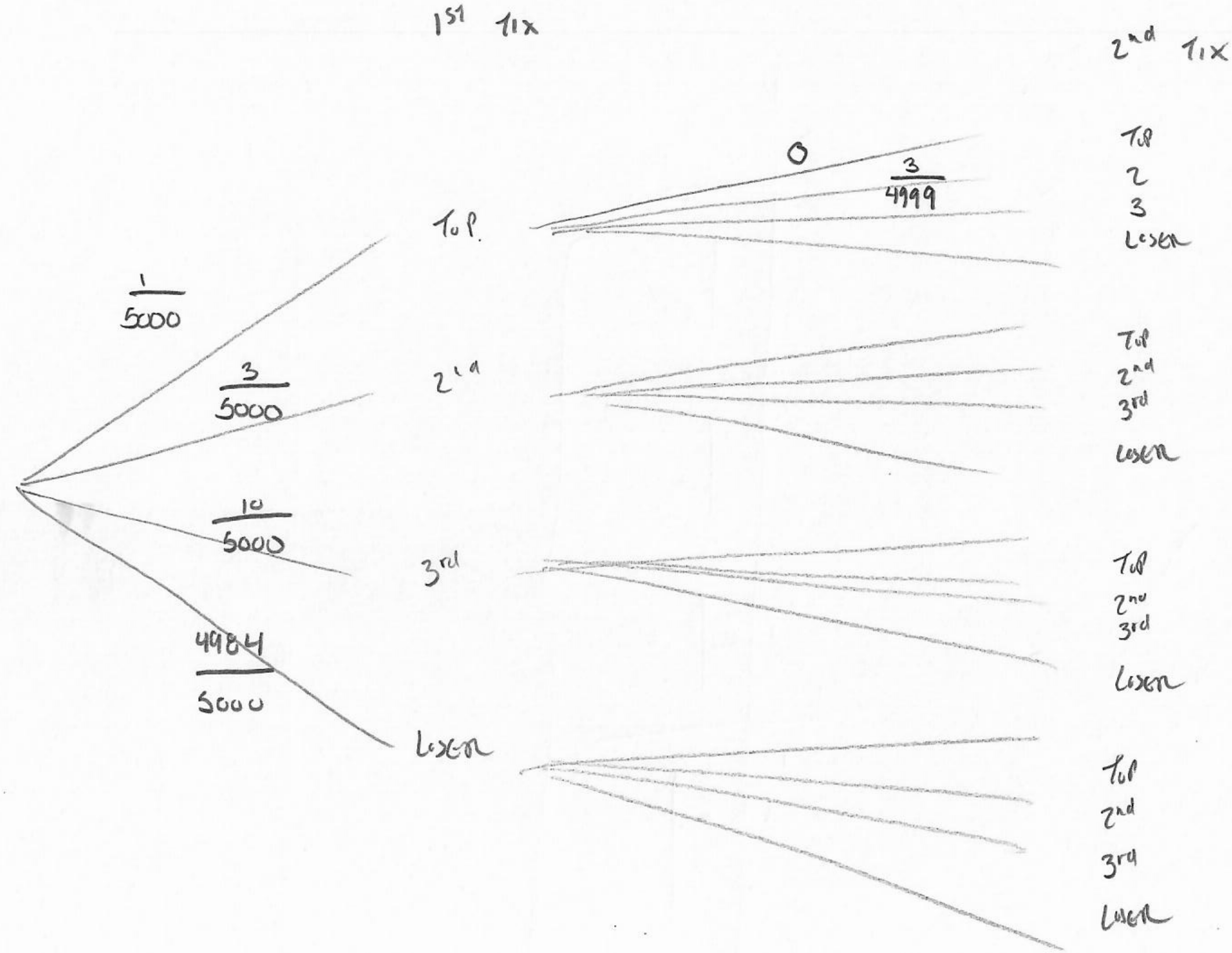
LET X = PROFIT ASSOCIATED WITH YOUR TICKET.

FIND PROBABILITY DISTRIBUTION FOR R.V. X.

x	998	498	73	-2
$p(x)$	$\frac{1}{5000}$	$\frac{3}{5000}$	$\frac{10}{5000}$	$\frac{4986}{5000}$

(PROBABILITIES MUST ADD UP TO 1)




1st Tix
2nd Tix
1/5000
3/5000
10/5000
4984/5000
Top
2nd
3rd
Loser
0
3/4999
Top
2
3
Loser
Top
2nd
3rd
Loser
Top
2nd
3rd
Loser
Top
2nd
3rd
Loser

REQUIREMENTS FOR PROB. FUNCTION $p(x)$

i) $0 \leq p(x) \leq 1$ (ALWAYS)

ii) SUM OF $p(x) = 1$

$$\left(\sum p(x) = 1 \right)$$

MEAN & STANDARD DEVIATION FOR DISCRETE RANDOM VARIABLE

MEAN FOR A DISCRETE RANDOM VARIABLE IS ALSO CALLED THE EXPECTED VALUE FOR A R.V.

IT'S THE VALUE YOU WOULD TO OBSERVE ON AVERAGE, IF THE EXPERIMENT WERE PERFORMED OVER & OVER AGAIN.

ex. RAFFL

ex. RAFFLE TIX:

x	998	498	73	-2
$p(x)$	$\frac{1}{5000}$	$\frac{3}{5000}$	$\frac{10}{5000}$	$\frac{4986}{5000}$

IMAGINE YOU PLAY THE RAFFLE 5,000 TIMES.

YOU PROFIT 998 ~ 1 TIME

YOU 498 ~ 3 TIMES

73 ~ 10 TIMES

-2 ~ 4986 TIMES

HOW MUCH DO YOU PROFIT ON AVERAGE?

$$\frac{(1)(998) + (3)(498) + (10)(73) + (4986)(-2)}{5000}$$

$$= \underbrace{\left(\frac{1}{5000}\right)(998) + \left(\frac{3}{5000}\right)(498) + \left(\frac{10}{5000}\right)(73) + \left(\frac{4986}{5000}\right)(-2)}_{\text{WEIGHTED AVERAGE OF ALL POSSIBLE VALUES FOR } x \text{, WEIGHTED BY } p(x)}$$

$$= \sum x\, p(x) = -1.35$$

EXPECTED VALUE FOR X (AVERAGE OF VALUE OF X)

IS -1.35.

IF WE PLAY OVER & OVER AGAIN,

ON AVERAGE, WE LOSE \$1.35

EACH TIME WE PLAY.

NOTATION: MEAN FOR A RANDOM VARIABLE X

IS DENOTED BOTH

SAME THING

$$\underbrace{\mu = E[X]}$$

EXPECTED VALUE

$$= \sum x p(x)$$

ex. GAME: YOU ROLL 2 DICE.

IF YOU GET DOUBLES: YOU WIN \$10.

IF YOU GET ONE ODD ONE EVEN: YOU LOSE \$10.

OTHERWISE: YOU WIN \$1.

FIND THE EXPECTED VALUE FOR THE AMOUNT YOU WIN/LOSE PLAYING THIS GAME. SHOULD YOU PLAY THIS GAME?

FIRST FIND PROBABILITY DISTRIBUTION.

X = AMOUNT WIN / LOSE (+ WIN , - LOSE)

X	10	-10	1
$p(x)$	$\frac{6}{36}$	$\frac{18}{36}$	$\frac{12}{36}$

1ST DIE

X	1	2	3	4	5	6
2nd DIE 1	10	-10	1	-10	1	-10
2	-10	10	-10	1	-10	1
3	1	-10	10	-10	1	-10
4	-10	1	-10	10	-10	1
5	1	-10	1	-10	10	-10
6	-10	1	-10	1	-10	10

MEAN / EXPECTED VALUE FOR X

$$\mu = E[X] = \sum x\,p(x)$$

SUM OF PRODUCTS $x\,p(x)$

$$(10)\left(\frac{6}{36}\right) + (-10)\left(\frac{18}{36}\right) + (1)\left(\frac{12}{36}\right)$$

$$= -3$$

INTERPRETATION: $E[X] = -3$

IF YOU PLAY REPEATEDLY, ON AVERAGE YOU EXPECT TO LOSE \$3 PER GAME.

IF YOU PLAY 100 TIMES, EXPECT TO LOSE \$300.

ex. GAME: ROLL 2 DICE:

IF YOU ROLL DOUBLES: YOU WIN A DOLLARS

IF YOU ROLL ONE ODD / ONE EVEN: YOU LOSE $10

OTHERWISE YOU WIN $1

PROB DISTR

x	A	-10	1
$p(x)$	$\frac{6}{36}$	$\frac{18}{36}$	$\frac{12}{36}$

FOR WHAT VALUE OF A WOULD YOU EXPECT TO BREAK EVEN, IF YOU PLAY REPEATEDLY?

i.e. FOR WHAT VALUE OF A IS $\underbrace{E[x] = 0}_{\text{"FAIR GAME"}}$?

$$\sum xp(x) = 0$$

$$\frac{6}{36}A + \frac{18}{36}(-10) + \frac{12}{36}(1) = 0$$

$$\frac{6}{36}A = \frac{180-12}{36} \rightarrow 6A = 180 - 12$$

$$A = \frac{180-12}{6} = 28$$

Def: LET X BE A DISCRETE RANDOM VARIABLE WITH PROBABILITY DISTRIBUTION $p(x)$ AND MEAN (EXPECTED VALUE) μ $(E[X])$. THE VARIANCE OF X IS

$$\sigma^2 = E[(x-\mu)^2]$$

$$= \sum (x-\mu)^2 p(x)$$

WHERE THE SUM IS TAKEN OVER ALL VALUES OF RAN. VAR. X.

ex.

X	10	-10	1
$p(x)$	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{3}$
$(x-\mu)^2$	13^2	$(-7)^2$	4^2

$\mu = -3$

$$\sigma^2 = \sum (x-\mu)^2 p(x)$$

$$= 13^2\left(\frac{1}{6}\right) + (-7)^2\left(\frac{1}{2}\right) + 4^2\left(\frac{1}{3}\right)$$

$$= 58 \longrightarrow \sigma = \sqrt{58} = 7.6158$$

Def: STAND. DEV. σ FOR RAND. VAR X IS $\sigma = \sqrt{\sigma^2}$

INTERPRETATION: YOU PLAY THE GAME OVER AND OVER.

EVERY TIME YOU PLAY, YOU GET A VALUE

FOR R.V. X = (WIN/LOSE)

$X_1 = 10$ $\quad X_4 = 1$ $\quad X_7 = -10$

$X_2 = -10$ $\quad X_5 = 1$ $\quad X_8 = -10$

$X_3 = 1$ $\quad X_6 = -10$ $\quad X_9 = 10$

ETC.

WHEN THIS LIST HAS ENOUGH #'S IN IT,

MEAN $\dfrac{\sum x_i}{N} = \sum x p(x) = \mu = E[X]$

$$\sigma = \frac{\sum (x_i - \mu)^2}{N} = \sum (x-\mu)^2 p(x)$$