4.82

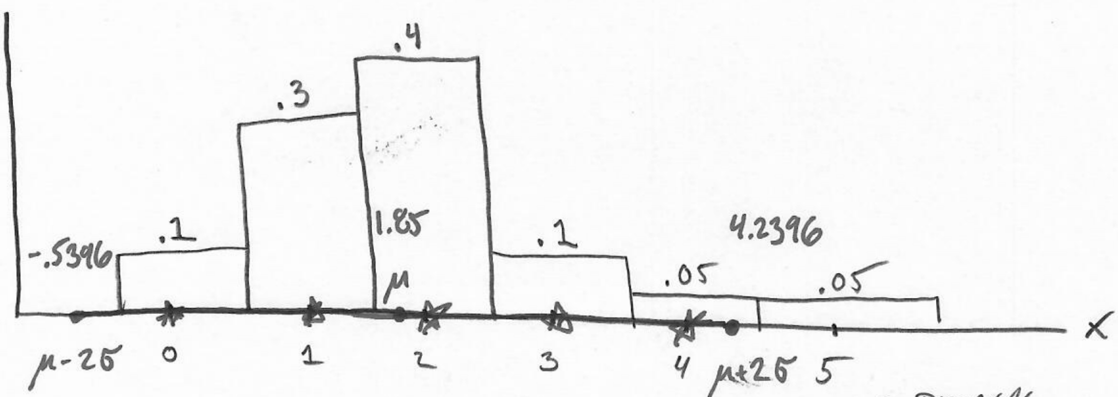
x	0	1	2	3	4	5
p(x)	.1	.3	.4	.1	.05	.05
$x-\mu$	-1.85	-.85	.15	1.15	2.15	3.15
$(x-\mu)^2$	3.4225	.7225	.0225	1.3225	4.6225	9.9225

$$\mu = \sum x p(x) = 0(.1) + 1(.3) + 2(.4) + 3(.1) + 4(.05) + 5(.05) = 1.85$$

$$\sigma^2 = \sum (x-\mu)^2 p(x)$$

$$\sigma^2 = (3.4225)(.1) + (.7225)(.3) + (.0225)(.4) + (1.3225)(.1) + (4.6225)(.05) + (9.9225)(.05) = 1.4275$$

$$\sigma = \sqrt{\sigma^2} = \sqrt{1.4275} = 1.1948$$



X DISCRETE SO BARS CENTERED ON INTEGERS

$$P(-.54 \le X \le 4.24) = P(x=0, x=1, x=2, x=3, x=4)$$

$$= p(0) + p(1) + p(2) + p(3) + p(4)$$

$$= 1 - p(5) = \boxed{.95}$$

C11

CHEVYCHEV'S THM:

THE PROPORTION OF MEASUREMENTS WITHIN $k\sigma$ OF μ

(i.e. $P(\mu - k\sigma \le X \le \mu + k\sigma)$) IS AT LEAST

$1 - \frac{1}{k^2}$.

$$\Rightarrow P(\mu - 2\sigma \le X \le \mu + 2\sigma) \ge 1 - \frac{1}{2^2} = \boxed{.75}$$

ex. A DRAWER CONTAINS 12 PENS.

7 PENS WORK, 5 PENS DON'T WORK.

YOU RANDOMLY SELECT 4 PENS.

LET X = NUMBER OF PENS THAT WORK. (DISCRETE RANDOM VARIABLE)

FIND PROBABILITY DISTRIBUTION FOR X, AND FIND THE EXPECTED VALUE $E[X]$.

PROBABILITY DISTRIBUTION

ALL POSSIBLE VALUES FOR X

X	0	1	2	3	4
p(x)	$\frac{C^7_0 C^5_4}{C^{12}_4}$	$\frac{C^7_1 C^5_3}{C^{12}_4}$	$\frac{C^7_2 C^5_2}{C^{12}_4}$	$\frac{C^7_3 C^5_2}{C^{12}_4}$	$\frac{C^7_4 C^5_0}{C^{12}_4}$
	.01	.14	.42	.35	.07

TOTAL # OF POSSIBLE OUTCOMES: SIZE OF SAMPLE SPACE

$${}_{12}C_4 = C^{12}_4 = \frac{12!}{4!\,(12-4)!} = 495 \quad \text{ALL EQUALLY LIKELY.}$$

$$p(3) = \frac{\overbrace{{}_7C_3}^{\text{PICK 3 WORKING PENS}} \times \overbrace{{}_5C_1}^{\text{PICK 1 BROKEN PEN}}}{{}_{12}C_4} = \frac{(35)(5)}{495} = .3535$$

MEAN / EXPECTED VALUE

$$\mu \qquad E[x] = \sum x\,p(x)$$

$$= 0(.01) + 1(.14) + 2(.42) + 3(.35) + 4(.07)$$

$$= \boxed{2.31}$$

AVERAGE VALUE X THAT WE SHOULD EXPECT IF THE EXPERIMENT IS REPEATED MANY TIMES.

§ 5.2 THE BINOMIAL PROBABILITY DISTRIBUTION

A PARTICULAR DISCRETE RANDOM VARIABLE

Def: A BINOMIAL EXPERIMENT IS ONE THAT SATISFIES THE FOLLOWING:

1. FIXED NUMBER (n) OF TRIALS, CALLED BINOMIAL TRIALS, ARE PERFORMED.
2. EACH TRIAL IS IDENTICAL. e.g. FLIP A COIN $\overset{n}{\overset{\downarrow}{3}}$ TIMES. EACH TRIAL IS A SINGLE COIN FLIP.
3. EACH TRIAL RESULTS IN ONE OF ONLY TWO POSSIBLE OUTCOMES.

   CONVENTION: SUCCESS & FAILURE
4. TRIALS ARE INDEPENDENT.

   THE OUTCOME OF A TRIAL IS NOT INFLUENCED BY THE OUTCOME OF ANY OTHER TRIAL.

5. $P(\text{SUCCESS}) = p$

$P(\text{FAILURE}) = q = 1-p$

TRUE FOR EVERY TRIAL

6. THE RANDOM VARIABLE X = # OF SUCESSES IN n TRIALS.

$x = 0, 1, 2, \ldots, n$

ex. SUPPOSE A BASKETBALL PLAYER MAKES 80% OF ALL FREE THROWS. THIS PLAYER SHOOTS 5 FREE THROWS.

LET X = # OF FREE THROWS THE PLAYER MAKES.

BINOMIAL EXPERIMENT: 5 IDENTICAL TRIALS (EACH FREE THROW) RESULTING IN EITHER SUCCESS OR FAILURE
(MAKE) (MISS)

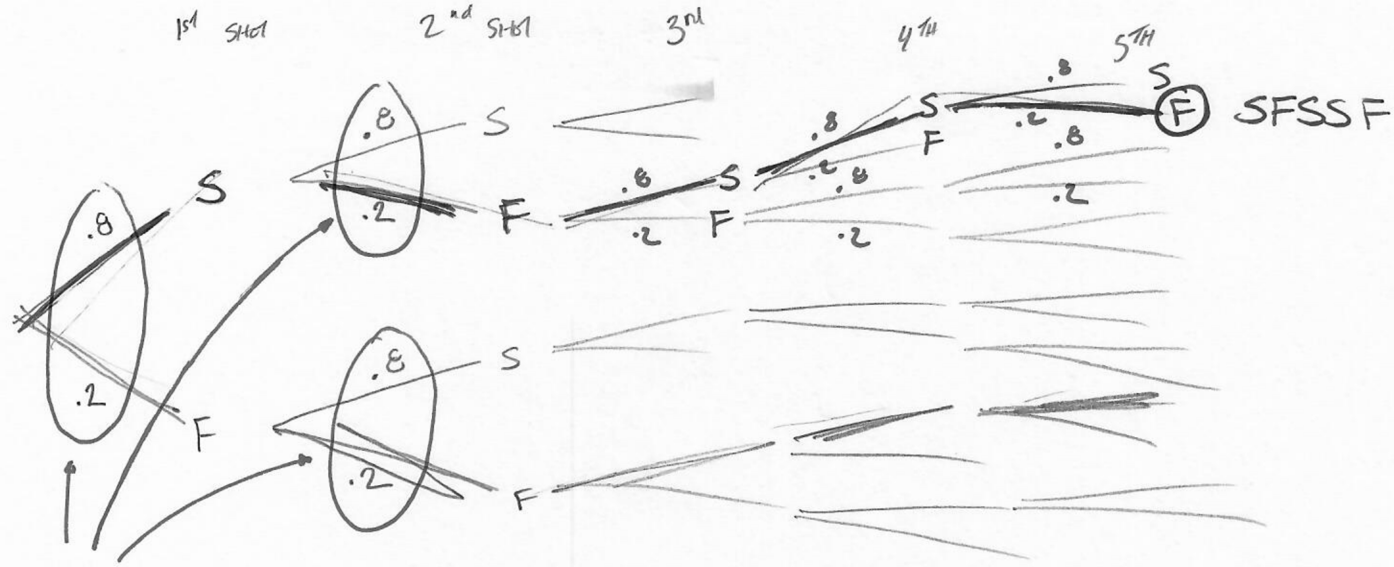
WE ASSUME ALL TRIALS (SHOT) ARE INDEPENDENT.

SUCCESS = MAKE SHOT $P(\text{SUCCESS}) = p = .8$

FAILURE = MISS SHOT $P(\text{FAILURE}) = q = 1-p = .2$

FIND PROBABILITY THAT THIS PLAYER MAKES EXACTLY 3 OF THE 5 SHOTS.

5 STAGES



p & q

DON'T CHANGE!

EACH TRIAL IS IDENTICAL!

INDEPENDENT!

(5 FREE THROWS)

IND. EVENTS.

$$P(\overbrace{S\,F\,S\,S}^{x=3}\,F) = P(S)P(F)P(S)P(S)P(F)$$

SIMPLE EVENT

$$= (.8)(.2)(.8)(.8)(.2) = (.8)^3(.2)^2$$

$$= \underline{.02048}$$

$$P(\overbrace{FFSSS}^{x=3}) = q\,q\,p\,p\,p$$

$$= (.2)(.2)(.8)(.8)(.8) = (.8)^3(.2)^2$$

$$= \underline{.02048}$$

3 SUCCESSES IN 5 TRIALS

$$P(X=3) = P(SSSFF) + P(SSFSF) + P(SSFFS) + \dots + P(FFSSS)$$

$$= \underline{{}_5C_3}\; p^3 q^2$$

$$P(x=3) = {}_5C_3\,(.8)^3(.2)^2 = 10(.8)^3(.2)^2 = \boxed{.2048}$$

$\underline{\ \checkmark\ }$ $\underline{\ \checkmark\ }$ $\underline{\qquad}$ $\underline{\ \checkmark\ }$ $\underline{\qquad}$

5 TRIALS

3 SUCCESSES

PICK 3 TRIALS FOR 3 SUCESSES

\# WAYS TO DO THIS ${}_5C_3$ OR C^5_3

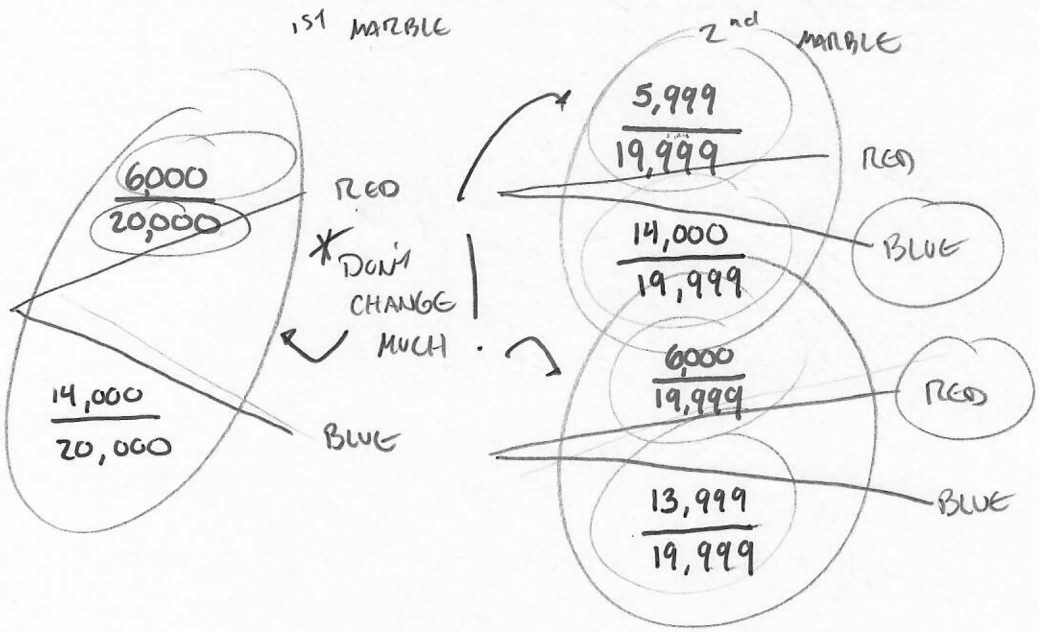
<u>Def</u>: GIVEN A BINOMIAL EXPERIMENT, WITH X = # SUCCESSES IN n TRIALS. X IS A DISCRETE RANDOM VARIABLE, CALLED A BINOMIAL RANDOM VARIABLE.

(FYI: "BINOMIAL EXP / R.V." = "BERNOULLI EXP / RV.")

$$P(x=k) = C^n_k \, p^k \, q^{n-k}$$

$$P(x=10) = C^{35}_{10} \left(\tfrac{1}{3}\right)^{10} \left(\tfrac{2}{3}\right)^{25} = \boxed{.1231}$$

ex. SUPPOSE A JAR CONTAINS 6,000 RED MARBLES
14,000 BLUE MARBLES.
YOU SELECT 2 MARBLES AT RANDOM.
FIND PROBABILITY THAT YOU SELECT 1 RED & 1 BLUE.



$$P(1R \,\&\, 1B) = P(RB) + P(BR)$$

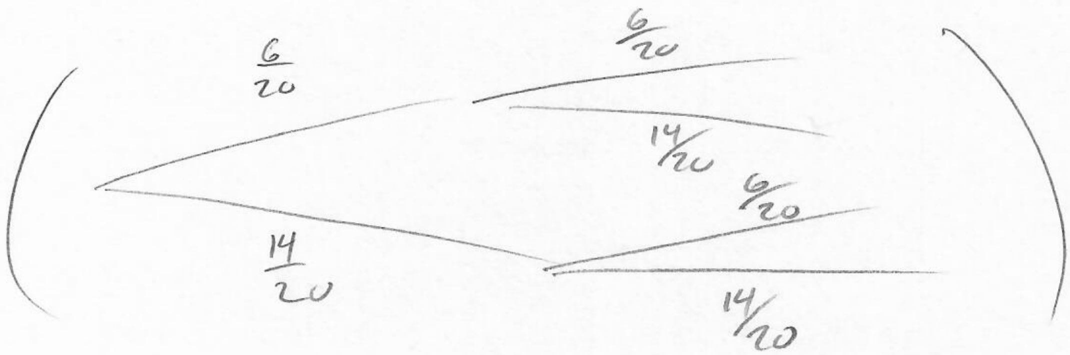
$$= \underbrace{\left(\frac{6000}{20000}\right)}_{P(R_1)}\underbrace{\left(\frac{14000}{19{,}999}\right)}_{P(B_2|R_1)} + \underbrace{\left(\frac{14{,}000}{20{,}000}\right)}_{P(B_1)}\underbrace{\left(\frac{6000}{19{,}999}\right)}_{P(R_2|B_1)}$$

MULTIPLICATION RULE.

$$= \underline{.4200210011}$$

THIS IS EXPERIMENT IS <u>APPROXIMATELY</u> BINOMIAL.

$\approx n = 2$ TRIALS IDENTICAL, INDEPENDENT



$p = \frac{6000}{2000}$ $\quad q = \frac{14000}{20000}$

(SUCCESS = RED FAILURE = BLUE)

PROBABILITY OF 1 RED & 1 BLUE? ← $x = 1$

LET x = # SUCCESSES (= # REDS) BINOMIAL R.V.

$$\boxed{P(x = k) = C^n_k \, p^k \, q^{n-k}}$$

$$P(x = 1) = C^2_1 \left(\frac{6000}{20000}\right)^1 \left(\frac{14000}{20000}\right)^1$$

$$= .42$$

GOOD APPROXIMATION.

$\boxed{\text{EXAM TUESDAY : 1.5, 2.2-4, 4.2-8}}$