EXAM 1: 1.5, 2.2 - 2.4, 4.2 - 4.8

10:30 - 12:10 6/23/2020

MEAN, MEDIAN, MODE,
VARIANCE, STAND. DEV.
- POPULATION
- SAMPLE

BLACK BOARD

↓

"CONTENT" IN LEFT SIDEBAR

"EXAM 1"

PROBABILITY
- SAMPLE SPACE,
  SIMPLE.
- FORMULAS (BASIC)

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$A = \{1, 3, 5, 7, 9\}$

$B = \{2, 4, 5, 6\}$

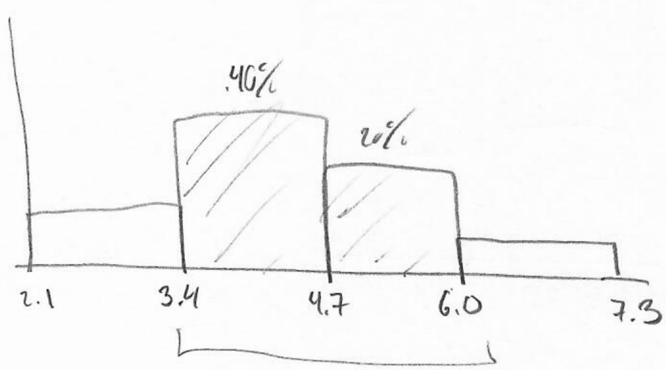
$A \cup B$? $A \cap B$?

ANSWERS: 4 - DECIMALS
WHEN NEC.

ex. 32

$\boxed{.0111}$ ± .0001

.01113456



- MUTUALLY EXCLUSIVE

- INDEPENDENT

P(A.

	A	A^c
B	.45	.25
B^c	.15	.15

$P(A)$? $P(A|B)$?

COUNTING: ${}_nC_r$, ${}_nP_r$. GENERAL mn RULE

CONDITIONAL PROB.

* LAW OF TOTAL PROB

* BAYES' RULE

DISCRETE RANDOM VARIABLES - EXPECTED VALUE. (§4.8)

$E[x] =$ ______________

$\sum x\,p(x)$ $\qquad$ $\sum (x-\mu)^2\,p(x)$

$\sum \frac{x_i}{n}$

12 PERSON COMMITEE NEEDS TO SELECT 5 MEMBERS :

PRES , V.PRES , SEC , TREAS. , REPRESENTATIVE

HOW MANY WAYS TO DO THIS?

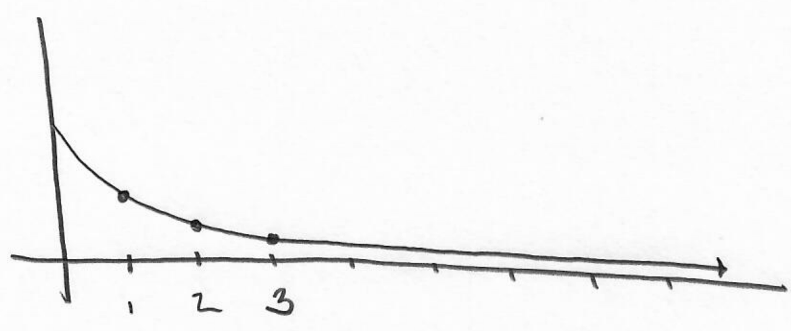
$${}_{12}C_5 \qquad {}_{12}P_5 \qquad {}_{5}C_{12} \qquad {}_{7}P_5$$

4.91 $p = 1$ (STRIKE OIL ON FIRST TRY) $p(1) = .1$

$p = 2$ (NO OIL , OIL) $\rightarrow$ $(.9)(.1) = .09$

$p = 3$ (NO OIL , NO OIL , OIL) $\rightarrow$ $(.9)(.9)(.1) = .081$

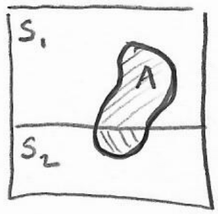
$p = x$ $\rightarrow$ $\underset{1}{(.9)}\underset{2}{(.9)}\ldots\underset{x-1}{(.9)}\underset{x}{(.1)} = .9^{x-1}(.1)$



$P(x = k)$

$P(x \leq k)$

(a) LAW OF TOTAL PROB: $\boxed{P(A) = P(A \cap S_1) + P(A \cap S_2)}$

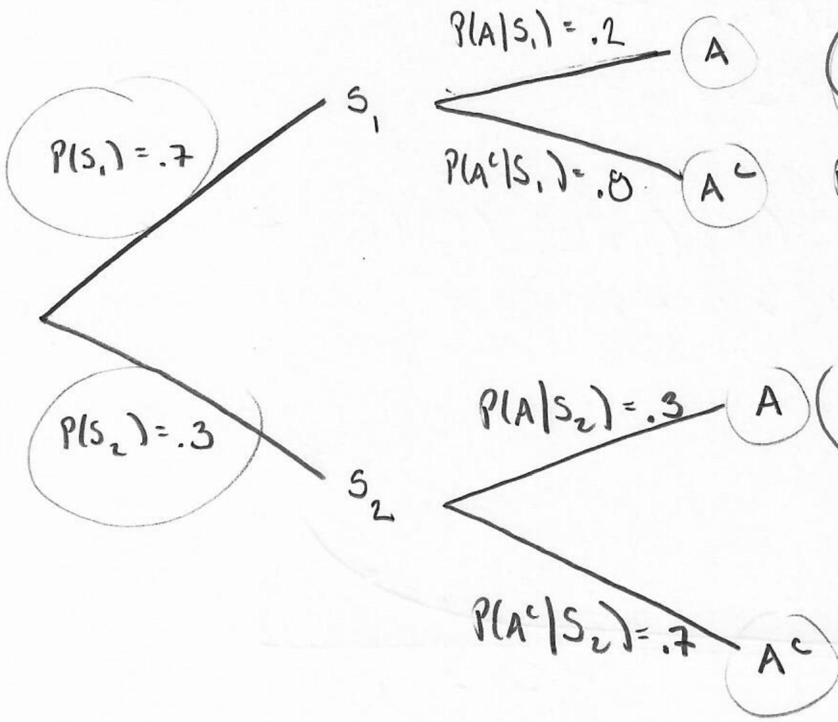


$$\boxed{= P(S_1)P(A|S_1) + P(S_2)P(A|S_2)}$$

$$= (.7)(.2) + (.3)(.3)$$

$$= .14 + .09 = \boxed{.23}$$

(b) BAYE'S RULE: $\boxed{P(S_1|A) = \dfrac{S(S_1)P(A|S_1)}{P(A)}} = \dfrac{(.7)(.2)}{(.23)} = \underline{\underline{.6087}}$



$P(S_1 \cap A) = (.7)(.2) = .14$

$P(S_1 \cap A^c) = (.7)(.8) = .56$

$P(S_2 \cap A) = (.3)(.3) = .09$

$P(S_2 \cap A^c) = (.3)(.7) = .21$

(a) $P(A) = .14 + .09 = .23$

(b) $P(S_1|A) = \dfrac{.14}{.14 + .09} = \underline{\underline{.6087}}$

§5.2 BINOMIAL RANDOM VARIABLES / §5.4 HYPERGEOMETRIC RANDOM VARIABLES

ex. EVERY TIME A COPY MACHINE MAKES A COPY THERE IS A PROBABILITY .01 THAT THE COPY MACHINE JAMS. IF YOU ARE MAKING 5 COPIES, FIND THE PROBABILITY DISTRIBUTION FOR $X = \underbrace{\#\text{ OF JAMS THAT OCCUR}}_{\#\text{ SUCCESSES}}$.

BINOMIAL: $n = 5$ TRIALS

SUCCESS = JAM → $P(\text{SUCCESS}) = p = .01$

FAILURE = NO JAM → $P(\text{FAILURE}) = q = .99$

$X = \#$ SUCCESSES

$$p(x=k) = {}_nC_k\, p^k q^{n-k}$$

x	0	1	2	3	4	5
$p(x)$	${}_5C_0(.01)^0(.99)^5$	${}_5C_1(.01)^1(.99)^4$	${}_5C_2(.01)^2(.99)^3$	${}_5C_3(.01)^3(.99)^2$	${}_5C_4(.01)^4(.99)^1$	${}_5C_5(.01)^5(.99)^0$
	.9501	.0480	.0010			

. . . . . .

ex.

90 MILLION REGISTERED REPUBLICAN VOTERS

85 MILLION REGISTERED DEMOCRATIC VOTERS

IF YOU SELECT 5 VOTERS FROM THE 175 MILLION REGISTERED VOTERS, FIND THE PROB. THAT EXACTLY 3 OF THEM ARE REGISTERED REPUBLICANS.

PRECISELY: ◦) # WAYS TO CHOOSE 5 REGISTERED VOTERS IS

$${}_{175,000,000}C_5 \quad \text{(ALL EQUALLY LIKELY)}$$

◦) # WAYS TO CHOOSE 3 REPUB (& 2 DEMOCRATS)

$$\underbrace{{}_{90,000,000}C_3 \cdot {}_{85,000,000}C_2}_{\text{2 STAGE EVENT}}$$

$\Rightarrow$ P(CHOOSE 3 REPUB, 2 DEM.) =

$$= \frac{{}_{90,000,000}C_3 \cdot {}_{85,000,000}C_2}{{}_{175,000,000}C_5} = .3209$$

APPROXIMATELY :

5 TRIALS : SELECTING A REGISTERED VOTER

ASSUME EACH TRIAL IS INDEPENDENT.

SUCCESS = REPUBLICAN $p = \frac{90 \text{ MILLION}}{175 \text{ MILLION}} = \frac{90}{175}$

FAILURE = DEMOCRAT $q = \frac{85 \text{ MILLION}}{175 \text{ MILLION}} = \frac{85}{175}$

BINOMIAL EXP. X = # REPUB.

$$P(X=k) = {}_nC_k \, p^k q^{n-k}$$

$$P(X=3) = {}_5C_3 \left(\frac{90}{175}\right)^3 \left(\frac{85}{175}\right)^2$$

$$= .3209$$ (SAME!)

PRECISE

ex. REPUBLICANS : 90

DEMOCRATS : 85

SELECT 5 VOTERS.

P(3 REPUB, 2 DEM)

$$= \frac{{}_{90}C_3 \cdot {}_{85}C_2}{{}_{175}C_5} = .3248$$

APPROXIMATE!

$n = 5$

$p = \frac{90}{175}$

$q = \frac{85}{175}$

$$P(3) = {}_5C_3 \left(\frac{90}{175}\right)^3 \left(\frac{85}{175}\right)^2$$

$$= .3209$$

NOT AS GOOD.

SUPPOSE A POPULATION OF N INDIVIDUALS IS DIVIDED INTO n_1 SUCCESSES & n_2 FAILURES.

SUPPOSE n INDIVIDUALS ARE CHOSEN AT RANDOM,

AND $X = \#$ SUCCESSES IN n INDIVIDUALS.

THEN X IS CALLED A HYPERGEOMETRIC RANDOM VARIABLE

AND $$P(X=k) = \frac{{}_{n_1}C_{\boxed{k}} \cdot {}_{n_2}C_{\boxed{n-k}}}{{}_{N}C_{\boxed{n}}}$$

$$n_1 + n_2 = N$$

$$k + (n-k) = n$$

APPROXIMATE HYPERGEOMETRIC R.V. WITH BINOMIAL R.V.

IF $\frac{n}{N} \le .05$ THEN X IS APPROXIMATELY BINOMIAL, AND $$P(X=k) \approx {}_{n}C_{k} \left(\frac{n_1}{N}\right)^{k} \left(\frac{n_2}{N}\right)^{n-k}$$

\# SELECTING < 5% OF POPULATION

$\Rightarrow$ PROB(SUCCESS) ON LAST TRIAL $\approx$ P(SUCCESS) ON ALL OTHER TRIALS

BECAUSE YOU ARE NOT REMOVING A SIGNIFICANT PROPORTION OF THE POPULATION

Pop N

$\swarrow$ $\searrow$

DEFECT. (SUCESS) n_1 NOT DEFECT. (FAILURES) n_2

ex. POPULATION OF COMP. CHIPS $N = 6$

DEFECTIVE: $n_1 = 2$

NOT DEFECTIVE: $n_2 = 4$

BEING CHOSEN: $n = 3$

$$P(X=k) = \frac{{}_{n_1}C_k \; {}_{n_2}C_{n-k}}{{}_{N}C_n}$$

X = # DEFECTIVE CHIPS

PROBABILITY DISTRIBUTION:

POSSIBLE VALUES OF X

$\frac{n}{N} = \frac{3}{6} = .5$

X	0	1	2
$p(x)$	$\frac{{}_2C_0 \, {}_4C_3}{{}_6C_3}$	$\frac{{}_2C_1 \, {}_4C_2}{{}_6C_3}$	$\frac{{}_2C_2 \, {}_4C_1}{{}_6C_3}$

NOW SUPPOSE $N = 600$ COMP. CHIPS

$n_1 = 200$ DEFECTIVE CHIPS

$n_2 = 400$ WORKING CHIPS.

SELECT 3 CHIPS.

$\frac{n}{N} = \frac{3}{600} = .005 < .05$

SUCCESS

USE BINOMIAL DISTRIBUTION TO APPROXIMATE $P($1 DEFECTIVE CHIP$)$

$p = \frac{200}{600}$ $q = \frac{400}{600}$

$n = 3$ TRIALS

$$P(x=k) = {}_nC_k \, (p)^k (q)^{n-k}$$

$$P(x=1) = {}_3C_1 \left(\frac{200}{600}\right)^1 \left(\frac{400}{600}\right)^2 = .4444$$