§5.2 BINOMIAL RANDOM VARIABLES

n TRIALS, IDENTICAL, INDEPENDENT

PROBABILITY OF SUCCESS p FOR EACH TRIAL

FAILURE $q = 1 - p$

X = # SUCCESS IN n TRIALS.

$$P(X = k) = {}_nC_k \, p^k q^{n-k}$$

MEAN / EXPECTED VALUE FOR BINOMIAL RAN. VAR. X

$$E[X] = \sum x\,p(x) = \sum_{x=0}^{n} x \underbrace{{}_nC_x \, p^x q^{n-x}}_{p(x)}$$

$$E[X] = np$$

e.g. EVERY EGG HAS A 7% CHANCE OF HAVING A DOUBLE YOKE.

IF YOU CRACK 20 EGGS AND COUNT X = # DOUBLE YOKES, FIND THE EXPECTED VALUE FOR X.

$n = 20$ TRIALS

SUCCESS = DOUBLE YOKE

$p = .07$

$q = .93$

$E[X] = np = (20)(.07)$

$= 1.4$

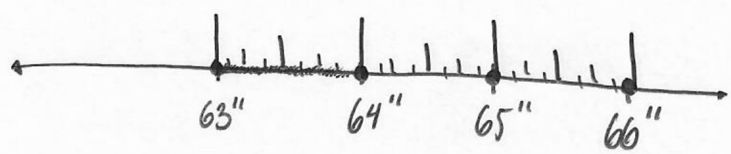
§6.1 PROBABILITY DISTRIBUTIONS FOR CONTINUOUS RANDOM VARIABLES

CONTINUOUS RANDOM VARIABLE ARE RANDOM VARIABLE (RULE THAT ASSIGNS A REAL # TO EVERY SIMPLE EVENT) AND IT IS CONTINUOUS (NOT DISCRETE!)

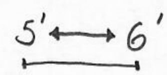
IT CAN TAKE ∞ MANY VALUES IN AN INTERVAL OF NUMBERS.

INFINITE PRECISION

∞ MANY DECIMALS.



e.g. HEIGHT OF HUMANS IS CONTINUOUS MEASUREMENT



\# SIBLINGS IS NOT CONTINUOUS

TEMPERATURE °F CONTINUOUS

WEIGHT, MASS, TIME

SINCE A CONTINUOUS RAN. VAR. CAN TAKE ∞ MANY VALUES, WE CANNOT ASSIGN POSITIVE PROBABILITIES $p(x)$ FOR EACH POSSIBLE VALUE OF X.

0 - 1 : ∞ MANY VALUES

EXPERIMENT: SELECTING ONE COFFEE BEAN AT RANDOM.

RANDOM VARIABLE X = WEIGHT (g)

8.103467320713400024...

8.1 g $8.05 \leq x < 8.15$

X CONTINUOUS

PROBABILITY DISTRIBUTION

LIST ALL POSSIBLE VALUES

x	0	.1	.08	.0023	...
$p(x)$	.	.	.	.	.

IMPOSSIBLE TO LIST.

IF WE PUT POSITIVE #'S HERE

THEN THEN THE SUM IS NO LONGER 1

(IT WOULD BE ∞)

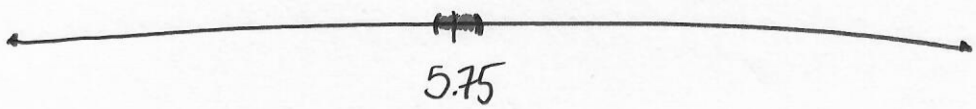
SUPPOSE X IS A CONTINUOUS RANDOM VARIABLE.

THEN $p(X = a) = 0$ (FOR ALL a).

EXPERIMENT: SELECT A RANDOM HUMAN

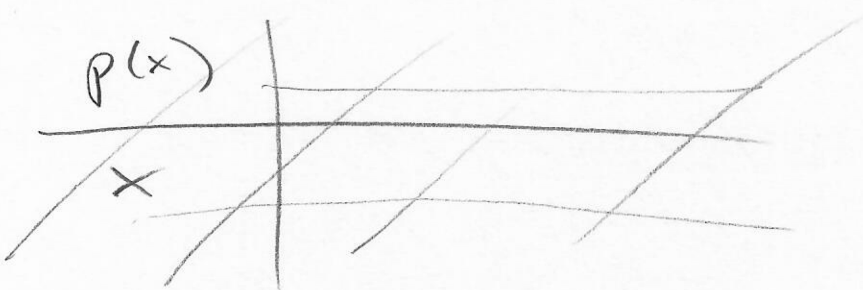
LET X = HEIGHT OF HUMAN (CONTINUOUS)

$$p(X = 5.75') = 0$$



$5.75\overline{0101010}1$ ← IS YOUR HEIGHT

$$P(X = 5.75\overline{01010101}) = 0$$

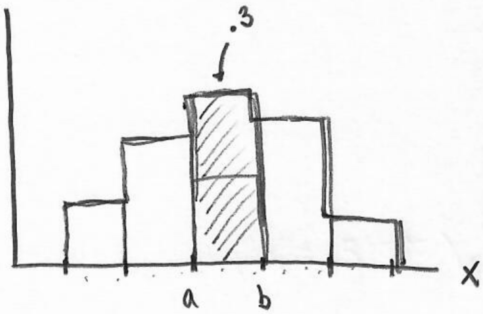


INSUFFICIENT FOR PROB. DIST. OF CONTINUOUS R.V.

NEW APPROACH:

MAKE A HISTOGRAM FOR CONT. RAN. VAR.

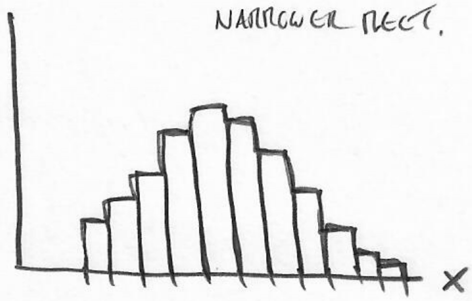
(SMALLER CLASSES)



$P(a \le X < b) = .3$

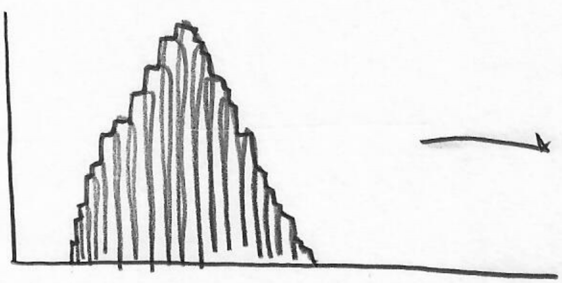
MORE → CLASSES

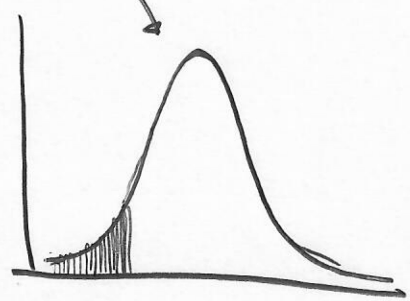
NARROWER RECT.



PROB. DISTRIBUTION OF CONTINUOUS RAN. VAR.

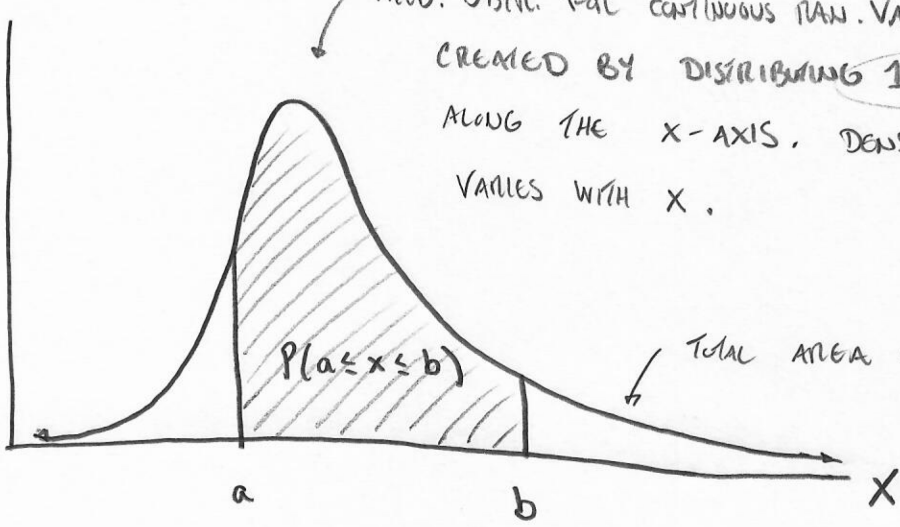
EXTREME →





PROB. DISTR. FOR CONTINUOUS RAN. VAR.

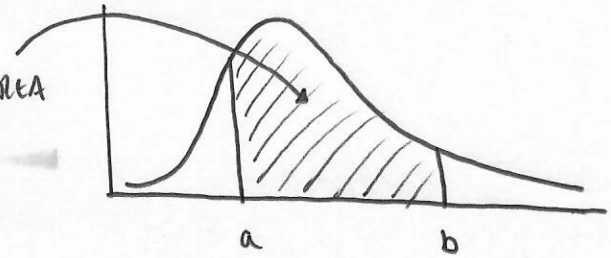
CREATED BY DISTRIBUTING 1 UNIT OF PROBABILITY ALONG THE X-AXIS. DENSITY OF PROBABILITY VARIES WITH X.



TOTAL AREA UNDER CURVE = 1.

▨ $= P(a \le x \le b)$

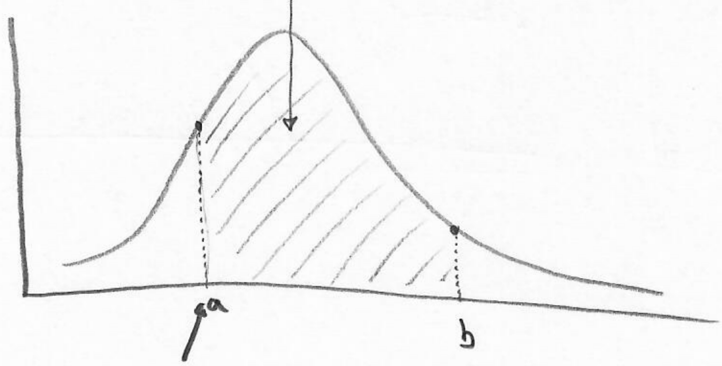
Note: $P(a \le x \le b)$ = AREA



$$\underbrace{P(x=a)}_{0} + P(a < x < b) + \underbrace{P(x=b)}_{0}$$

X CONTINUOUS

$$\therefore \boxed{P(a \le x \le b) = P(a \le x < b) = P(a < x \le b) = P(a < x < b)}$$



$P(x=a)=0$

$P(x=b)=0$

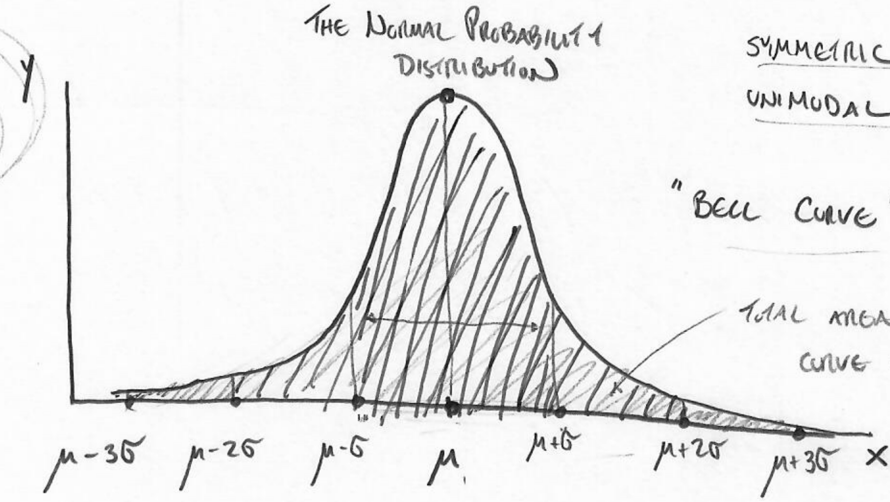
No WIDTH $\Rightarrow$ AREA 0

§6.2 THE NORMAL PROBABILITY DISTRIBUTION — FOR CONTINUOUS R.V.

THE NORMAL PROB. DISTR. IS A PARTICULAR DISTRIBUTION THAT SOME CONTINUOUS RANDOM VARIABLES HAVE, AND MANY CONTINUOUS RANDOM VARIABLE HAVE APPROXIMATELY.

$$y = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

σ, μ PARAMETERS



SYMMETRIC

UNIMODAL

"BELL CURVE"

AREA = $P(\mu-\sigma \le x \le \mu+\sigma) = .6827$

AREA = $P(\mu-2\sigma \le x \le \mu+2\sigma) = .9545$

AREA = $P(\mu-3\sigma \le x \le \mu+3\sigma) = .9973$

WE GET DIFFERENT NORMAL DISTRIBUTION FOR DIFFERENT VALUES OF μ & σ.

- DISTRIBUTION IS CENTERED AT μ
- LARGER VALUE FOR $\sigma \rightarrow$ WIDER, SHORTER DISTRIBUTION

  SMALLER VALUE FOR $\sigma \rightarrow$ NARROWER, TALLER DISTRIBUTION