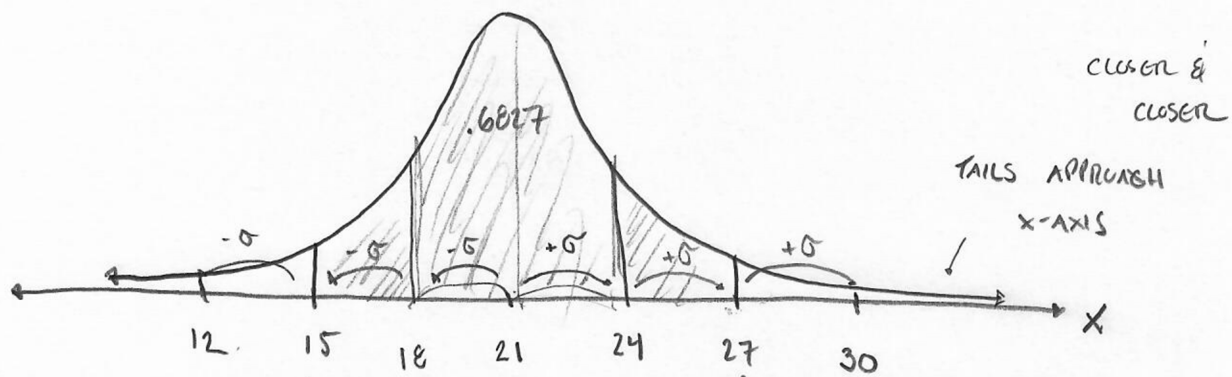
§ 6.2 CONTINUED

ex. SKETCH A NORMAL PROBABILITY DISTRIBUTION FOR A R.V. X WITH MEAN $\mu = 21$, AND STANDARD DEVIATION $\sigma = 3$.

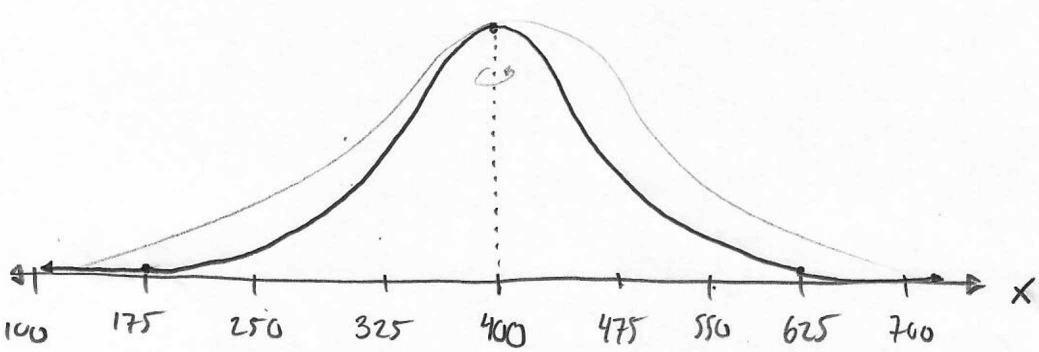


AREA BETWEEN 18 & 24 $(\mu - \sigma, \mu + \sigma)$

$= P(18 \leq X \leq 24) = \underline{.6827}$

$P(15 \leq X \leq 27) = \underline{.9545}$

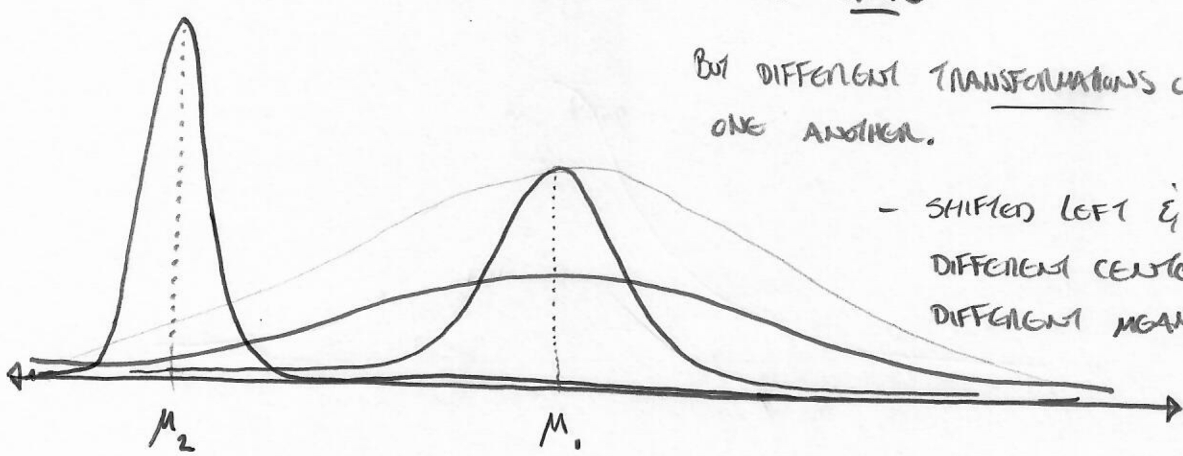
ex. SKETCH NORMAL PROBABILITY DISTRIBUTION WITH MEAN $\mu = 400$, & STAND. DEV. $\sigma = 75$.



§6.3 TABULATED AREAS UNDER THE NORMAL PROBABILITY DISTRIBUTION.

ALL NORMALL PROB. DISTR.'S HAVE THE SAME SHAPE

BUT DIFFERENT TRANSFORMATIONS OF ONE ANOTHER.



- SHIFTED LEFT & RIGHT
  DIFFERENT CENTERS
  DIFFERENT MEANS

- STRETCHED : STRETCHED HORIZONTALLY (AREA BELOW ALL IS 1)

  $\updownarrow$

  COMPRESSED VERTICALLY

  OR COMPRESSED HORIZONTALLY

  $\updownarrow$

  STRETCHED VERTICALLY

A NORMAL RANDOM VARIABLE X IS STANDARDIZED BY EXPRESSING ITS VALUE AS THE NUMBER OF STANDARD DEVIATIONS σ IT ~~IS~~ LIES ABOVE (+) OR BELOW (-) THE MEAN μ. (A NEW UNIT)

$$X \xrightarrow{\text{STANDARDIZE}} \frac{X-\mu}{\sigma}$$

σ ABOVE OR BELOW MEAN.

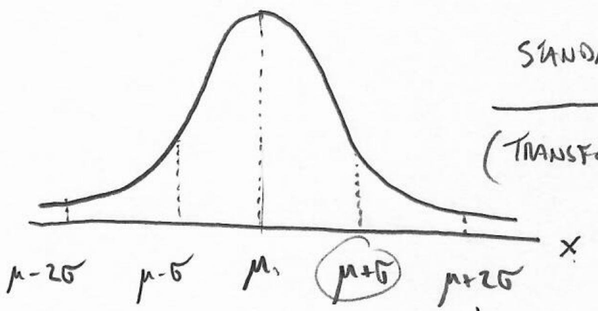
e.g. $\mu = 35$, $\sigma = 8$

$$X = 41 \xrightarrow{\text{STANDARDIZE}} \frac{X-\mu}{\sigma} = \frac{41-35}{8} = \frac{6}{8} = .75$$

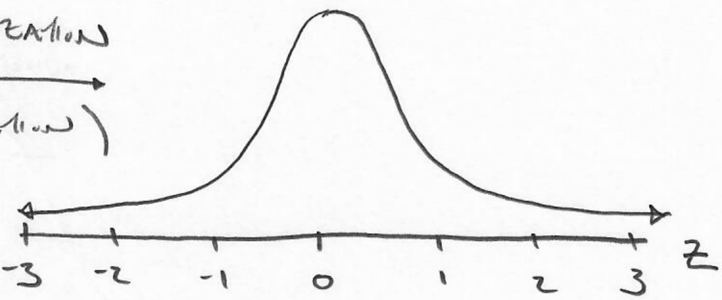
STANDARDIZATION

THIS CHANGE IN UNITS $X \longrightarrow Z = \dfrac{X - \mu}{\sigma}$

HAS AN EFFECT ON THE PROBABILITY DISTRIBUTION



$\xrightarrow[\text{(TRANSFORMATION)}]{\text{STANDARDIZATION}}$



↳ -2 ↳ -1 ↳ 2

$Z = \dfrac{X-\mu}{\sigma} = \dfrac{\mu+\sigma-\mu}{\sigma} = 1$

$Z = \dfrac{X-\mu}{\sigma} = \dfrac{\mu-\mu}{\sigma} = 0$

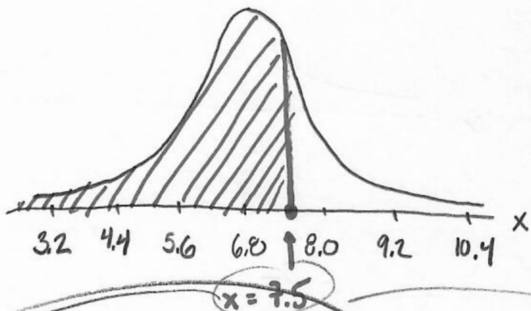
BEGIN WITH NORMAL R.V. X WITH MEAN μ & S.D. σ $\xrightarrow{\text{STANDARDIZE}}$ $Z = \dfrac{X-\mu}{\sigma}$ STANDARD NORMAL R.V. Z WITH MEAN $\mu = 0$ & S.D $\sigma = 1$.

AREAS UNDER THIS NORMAL PROB. DISTR HAVE BEEN TABULATED.

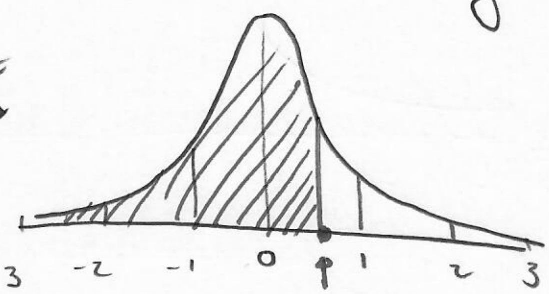
THEY'VE BEEN CALCULATED AND LISTED IN THE FORM OF A TABLE.

e.g. Suppose the mass of individual coffee beans X is a normally distributed random variable with mean $\mu = 6.8$ g and S.D. $\sigma = 1.2$ g.

Find the probability that a coffee bean has mass ≤ 7.5 g.



STANDARDIZE $\longrightarrow$ $z = \frac{x - \mu}{\sigma}$



$P(x \leq 7.5)$ = AREA ▨

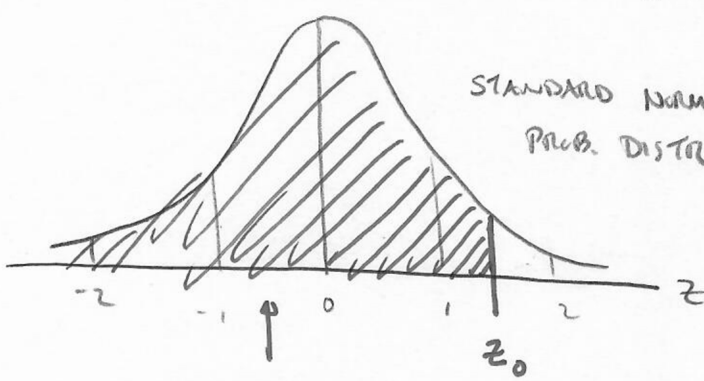
$$z = \frac{x - \mu}{\sigma} = \frac{7.5 - 6.8}{1.2} = \frac{.7}{1.2} = .5833$$

$\updownarrow$

AREA ▨ = $P(z \leq \underline{.5833})$

FIND IN TABLE
READ PROBABILITY

This can be looked up in a table!



STANDARD NORMAL PROB. DISTR

$P(x \leq 7.5)$

$= P(z \leq .58)$

$= \boxed{.7190}$

AREA $= P(z \leq z_0)$ = Find z_0 in table & read probability.

ex. SUPPOSE A RANDOM VARIABLE X IS NORMALLY DISTRIBUTED WITH MEAN $\mu = 72$ AND STAND. DEV. $\sigma = 16$

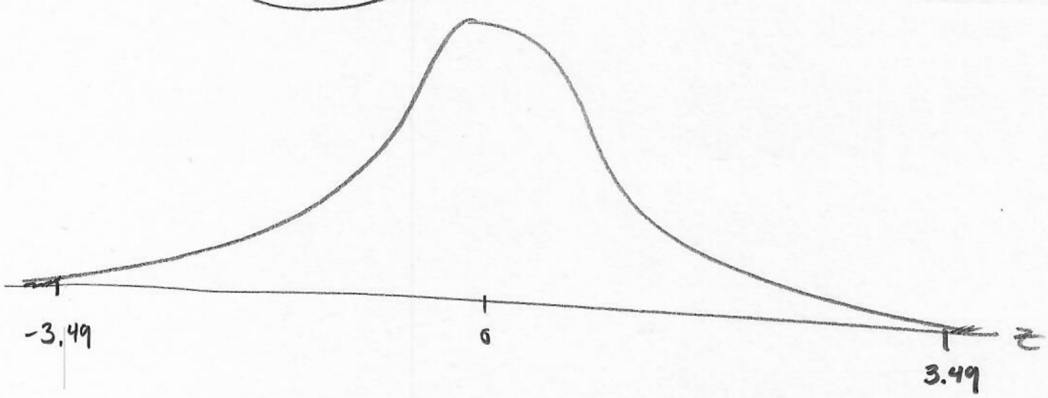
(a) FIND $P(X \le 67)$

STANDARDIZE: $z = \dfrac{X - \mu}{\sigma}$

$$= P\left(z \le \frac{67 - 72}{16}\right) = P(z \le -.31)$$

LOOK UP IN TABLE!

$$= .3783$$



$P(z \le -3.49) \approx 0$

$P(z \le 3.49) \approx 1$

COMPLIMENTARY EVENTS

(b) FIND $P(X \ge 67) = 1 - P(X < 67)$

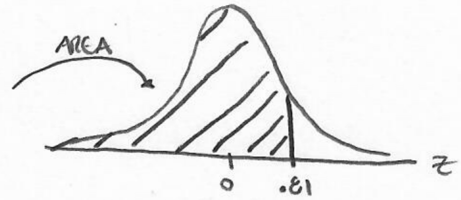
$$= 1 - P(X \le 67)$$

$$= 1 - .3783 = \underline{\underline{.6217}}$$

$$z = \frac{x-\mu}{\sigma}$$

(c) FIND $P(x \le 85) = P\left(z \le \frac{85-72}{16}\right)$

$$= P(z \le .81)$$



$$= .7910$$

(d) FIND $P(67 \le x \le 85)$

$$P\left(\frac{67-72}{16} \le z \le \frac{85-72}{16}\right)$$

$$P(-.31 \le z \le .81)$$

BETWEEN -.31 & .81 MEANS LEFT OF .81 AND NOT LEFT OF -.31

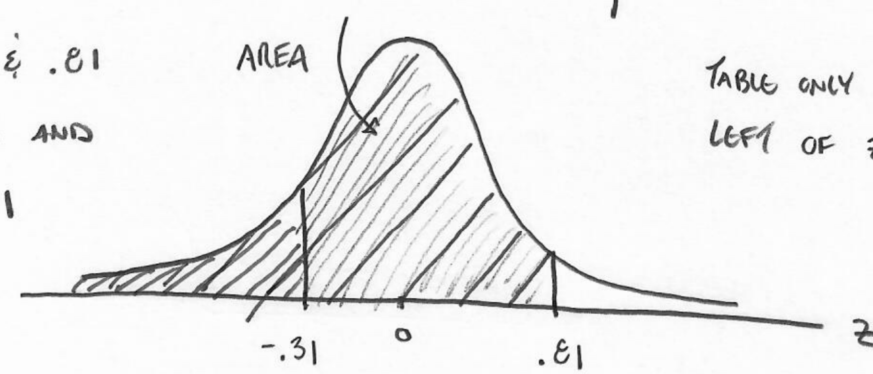


TABLE ONLY GIVES AREA LEFT OF z_0.

$$P(z \le .81) - P(z \le -.31)$$

$$= .7910 - .3783$$

$$= .4127$$

R.V. X

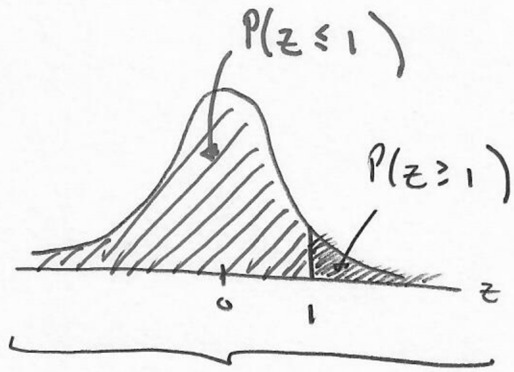
ex. STUDIES SHOW THAT FUEL ECONOMY (M.P.G.) FOR COMPACT CARS IS NORMALLY DISTRIBUTED WITH MEAN $\mu = 35.5$ MPG & STAND DEV. $\sigma = 4.5$ M.P.G.

WHAT PERCENTAGE OF COMPACT CARS GET 40 MPG OR MORE?

$$P(X \geq 40) = P\left(Z \geq \frac{40 - 35.5}{4.5}\right) =$$

$$Z = \frac{X - \mu}{\sigma}$$

$$= P(Z \geq 1)$$



ADD UP TO 1

(TOTAL AREA UNDER CURVE = 1)

$$= 1 - P(Z \leq 1)$$

TABLE

$$= 1 - .8413 =$$

$$= \boxed{.1587}$$

ex. FUEL ECONOMY OF COMP. CARS

$\mu = 35.5$

$\sigma = 4.5$

WHAT FUEL ECONOMY MUST A CAR GET (MPG) IN ORDER TO BE IN THE 95TH PERCENTILE FOR FUEL ECONOMY OF COMP. CARS?

Def: A MEASUREMENT x_0 IS IN THE N^{TH} PERCENTILE IF $P(x \le x_0) = N\% = \frac{N}{100}$.

FOR WHAT VALUE x_0 IS $P(x \le x_0) = .95$?

FOR WHAT VALUE x_0 IS $P\left(z \le \frac{x_0 - \mu}{\sigma}\right) = .95$

FIND PROB IN BODY OF TABLE (OR CLOSEST WE CAN FIND)

DETERMINE THIS # Z-VALUE

$P(z \le 1.64) = .9495$

$P(z \le 1.645) \approx .95$ EXACTLY IN THE MIDDLE!

$P(z \le 1.65) = .9505$

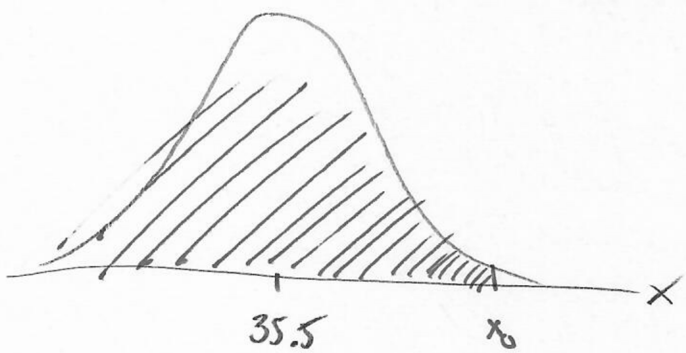
$$\frac{x_0 - \mu}{\sigma} = \boxed{\frac{x_0 - 35.5}{4.5} = 1.645}$$

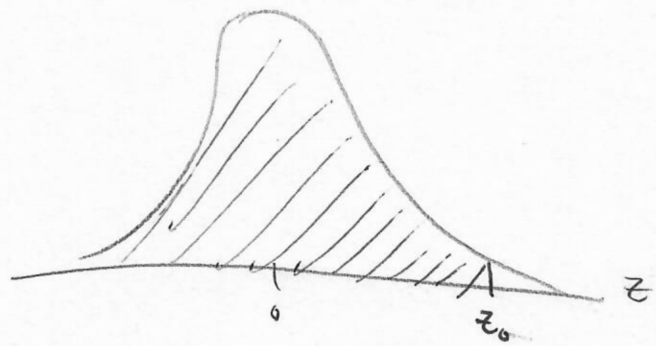
SOLVE FOR x_0.

$$\cancel{4.5}\left(\frac{x_0 - 35.5}{\cancel{4.5}}\right) = 1.645 \times 4.5$$

$$x_0 - \cancel{35.5} = 1.645 \times 4.5$$
$$+\cancel{35.5} \qquad\qquad + 35.5$$

$$X_0 = (1.645)(4.5) + 35.5 = \boxed{42.9025 \text{ MPG}}$$
$$(z_0)(\sigma) + \mu$$





$P(x \le x_0) = .95$

$\downarrow$

$\underline{\underline{X_0 = ?}}$

$P(z \le z_0) = .95$

$\downarrow$ TABLE

$\underline{\underline{z_0 = 1.645}}$

$$z_0 = \frac{x_0 - \mu}{\sigma} \quad \text{(STANDARDIZATION)}$$

SOLVE FOR x_0

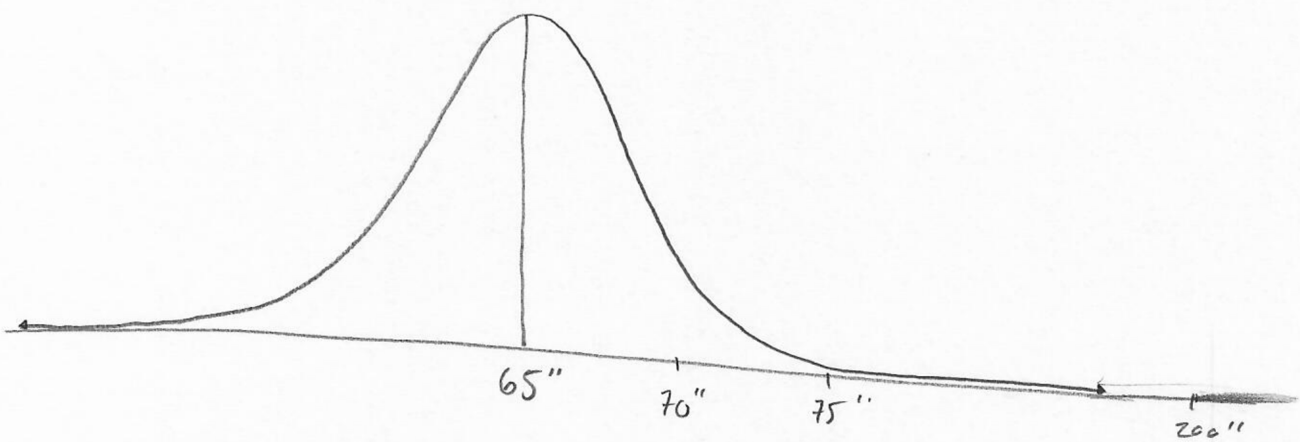
$$\boxed{x_0 = z_0 \sigma + \mu}$$

X , Z RANDOM VARIABLE

X_0 , Z_0 ARE SPECIFIC VALUES FOR X , Z

$$P(X \le X_0) = P(Z \le Z_0)$$

WHERE $Z_0 = \dfrac{X_0 - \mu}{\sigma}$



$P(X \ge 200") = .00000000000001$