EXAM 1 - REVIEW ON BLACKBOARD

CLICK ON EXAM GRADE → GRADE DETAILS

→ VIEW ATTEMPT

(- IF YOUR ANSWER TO ONE QUESTION CAUSES YOU TO GET ANOTHER QUESTION WRONG EMAIL ME TO LET ME KNOW.)

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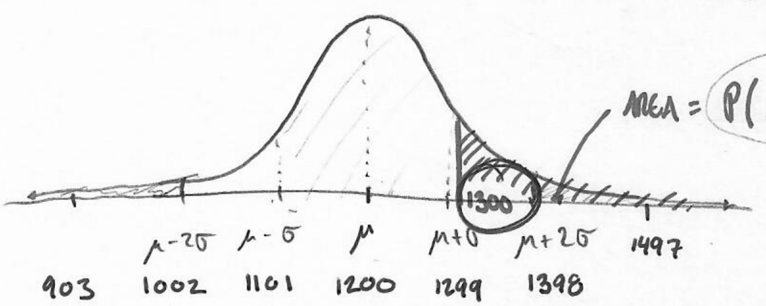
§6.3 AREA UNDER NORMAL DISTRIBUTION WITH TABLES

EXPERIMENT: SELECT 8 PEOPLE AT RANDOM

↳ RANDOM VARIABLE X = TOTAL WEIGHT (CONT.)

GIVEN: X HAS A NORMAL PROB. DISTR.



AREA $= P(X \geq 1300) = P(X > 1300) + \underbrace{P(X = 1300)}_{0}$

$P(X \geq 1300)$

$= P\left(Z \geq \frac{1300 - 1200}{99}\right)$

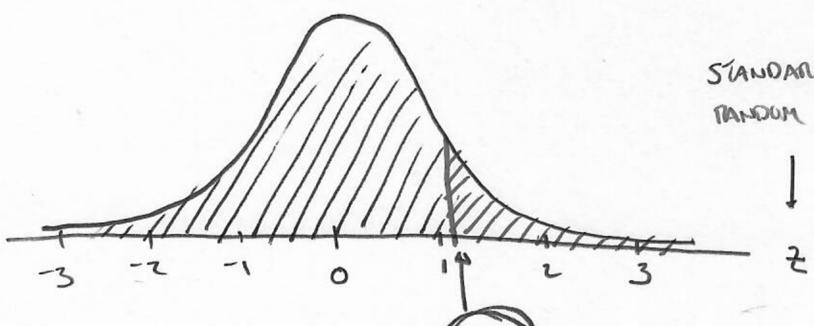
$Z = \frac{X - \mu}{\sigma}$

$= P(Z \geq 1.01)$

STANDARD NORMAL RANDOM VARIABLE

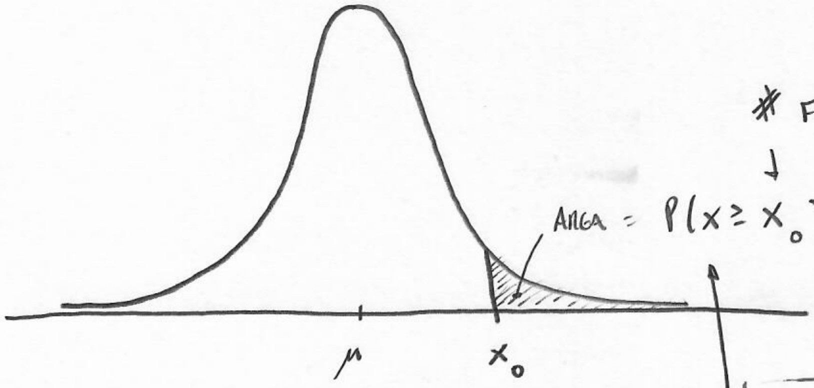
↓ $\mu = 0$

$\sigma = 1$



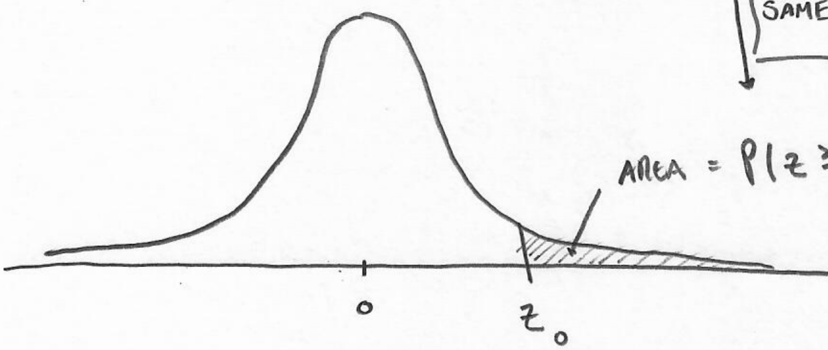
$P(Z \geq 1.01) = 1 - P(Z \leq 1.01) = 1 - .8438 = \boxed{.1562}$

UNITS : # σ ABOVE / BELOW MEAN



SAME WHEN $z_0 = \dfrac{x_0 - \mu}{\sigma}$

STANDARDIZATION



WE HAVE TABLE FOR $P(Z \leq z_0)$

ANY # BETWEEN $-3.49 \leq z \leq 3.49$

$$P(X \geq 1300) = P\left(Z \geq \frac{1300-1200}{99}\right) = 1 - P\left(Z \leq \frac{1300-1200}{99}\right)$$

STANDARDIZE

$$= \boxed{.1562}$$

PROBABILITY THAT X IS ≥ 1300

PROB. THAT TOTAL WEIGHT OF 8 PEOPLE EXCEEDS 1300 LBS IS .1562

15.62%

$$\boxed{P(x \geq 1500)} = P\left(z \geq \frac{1500 - 1200}{99}\right)$$

$$z = \frac{x - \mu}{\sigma}$$

$$= 1 - \underbrace{P\left(z \leq \frac{1500 - 1200}{99}\right)}$$

(TABLE: LOOK UP $\frac{300}{99} = 3.03$

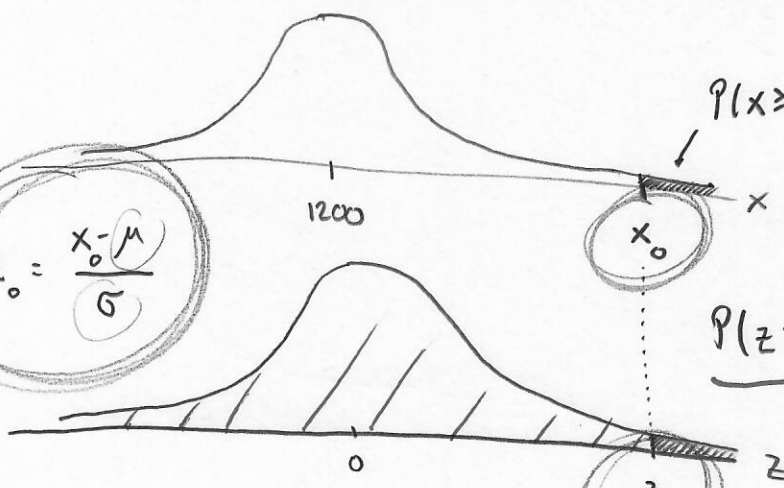
$$= 1 - .9988 = \boxed{.0012}$$

0.12%

$$P(x \geq 1500) = .0012$$

HOW MUCH WEIGHT SHOULD THE ELEVATOR BE ABLE TO HOLD IF THE PROBABILITY THAT THE WEIGHT OF 8 PEOPLE EXCEEDS THIS VALUE IS TO BE .0005.

$P(x \geq x_0) = .0005$



CHANGE OF UNITS

$$z_0 = \frac{x_0 - \mu}{\sigma}$$

x_0

FIND z_0 KNOWN PROB

$$\underbrace{P(z \geq z_0) = .0005}$$

$$P(z \leq z_0) = .9995$$

z_0

z_0 3.27

$$z_0 = \frac{x_0 - 1200}{99} \longrightarrow 3.27 = \frac{x_0 - 1200}{99}$$

$$(99)(3.27) = x_0 - 1200$$

$$1200 + (99)(3.27) = x_0 = \boxed{1523.73}$$

$$\left(P(x \geq 1523.73) = .0005 \right)$$

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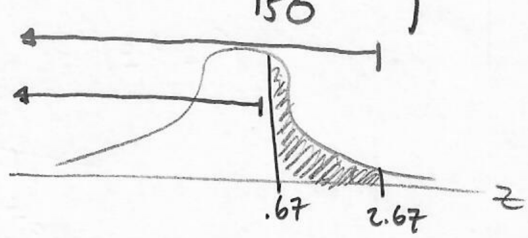
ex. SCORES ON SAT ARE APPROX. NORMALLY DISTRIBUTED WITH MEAN 800 AND STANDARD DEV. 150.

(a) FIND THE PROB THAT A RANDOMLY SELECTED STUDENT SCORES BETWEEN 900 AND 1200.

(b) WHAT SCORE MUST A STUDENT GET IN ORDER TO BE IN THE TOP 1% (99TH PERCENTILE).

(a) $P(\underline{\underline{900}} \leq \underline{\underline{x}} \leq \underline{\underline{1200}}) = P\left(\frac{900-800}{150} \leq z \leq \frac{1200-800}{150}\right)$

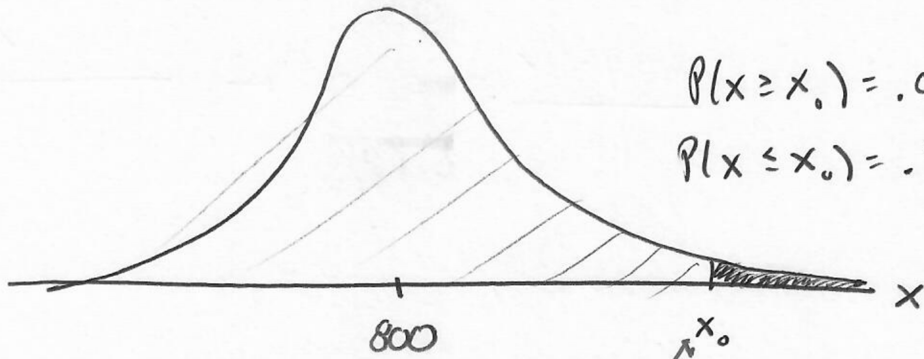
$$\left(z = \frac{x - \mu}{\sigma} \right)$$



$$= P(.67 \leq z \leq 2.67) = P(\underline{z \leq 2.67}) - P(\underline{z \leq .67})$$

$$= .9962 - .7486 = .2476$$

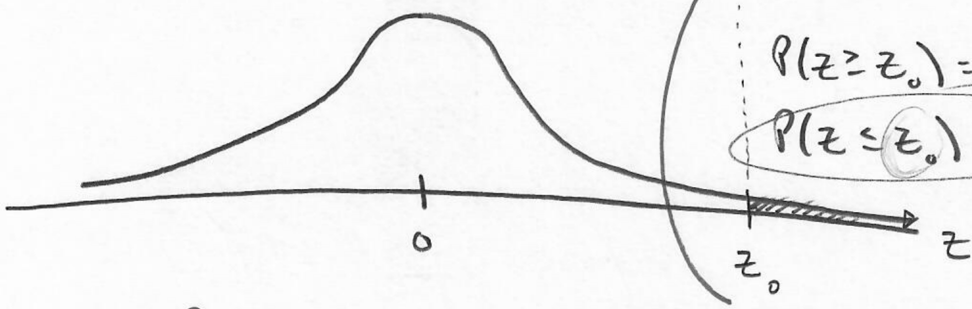
(b)



$P(x \geq x_0) = .01$

$P(x \leq x_0) = .99$

$$z_0 = \frac{x_0 - \mu}{\sigma}$$



$P(z \geq z_0) = .01$

$P(z \leq z_0) = .99$

2.33

$P(z \leq z_0) = .99$

PROBABILITY

FIND THIS PROB. IN BODY OF TABLE & THEN FIND z_0 IN ROW/COL.

$z_0 = 2.33$

$$2.33 = \frac{x_0 - 800}{150}$$

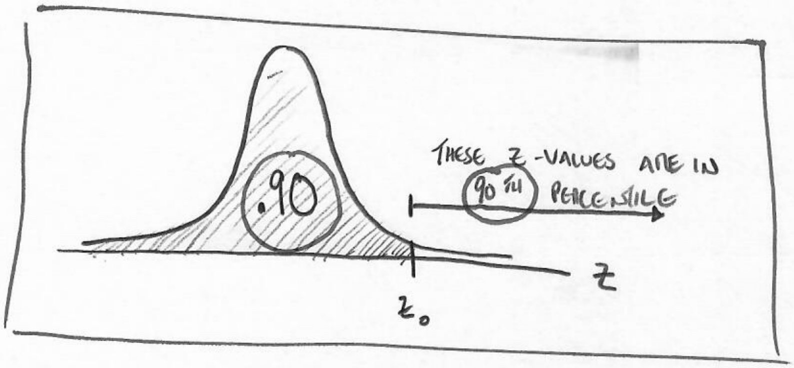
$2.33(150) = x_0 - 800$

$800 + 2.33(150) = x_0$

$1{,}149.5 = x_0$

YOU NEED TO SCORE AT LEAST 1,149.5 (1150) TO BE IN TOP 1%.

6.11



90^{TH} PERCENTILE $= [z_0, \infty)$

$z \geq z_0$

FIND $z_0 = 1.28$

WHERE $P(z \leq z_0) = .90$

ALL VALUES $z \geq 1.28$ ARE IN THE 90^{TH} PERCENTILE.

... IN THE N^{TH} PERCENTILE $\equiv$ ... IN THE TOP $(100-N)$ PERCENT

e.g. 95^{TH} 5

HW §6.3 SOON!

§6.4 THE NORMAL APPROXIMATION TO THE BINOMIAL PROBABILITY DISTRIBUTION

LET X BE BINOMIAL RANDOM VARIABLE (DISCRETE)

WITH n TRIALS, EACH WITH PROBABILITY OF SUCCESS p

& PROB. OF FAILURE q. X = # SUCCESSES IN n TRIALS.

WE KNOW $\boxed{P(X=k) = C^n_k \, p^k \, q^{n-k}}$ PROB DISTR.

FOR $k = 0, 1, 2, \ldots, n$.

WHAT IF WE WANT TO KNOW $P(X \leq k)$?

e.g. ROLL A DIE 20 TIMES & COUNT X = # SIXES.

$$P(X \leq 10) = P(X=0) + P(X=1) + P(X=2) + \ldots + P(X=10)$$

$$= C^{20}_0 \left(\tfrac{1}{6}\right)^0 \left(\tfrac{5}{6}\right)^{20} + C^{20}_1 \left(\tfrac{1}{6}\right)\left(\tfrac{5}{6}\right)^{19} + C^{20}_2 \left(\tfrac{1}{6}\right)^2 \left(\tfrac{5}{6}\right)^{18} + \ldots$$

$$+ C^{20}_{10} \left(\tfrac{1}{6}\right)^{10} \left(\tfrac{5}{6}\right)^{10}$$

CUMULATIVE BINOMIAL PROBABILITY. $P(X \leq k)$

CALCULATING CUMULATIVE BIN. PROB. DIST. IS TEDIOUS.

ex. X IS BIN. R.V.

$n = 20$, $p = .7$, $q = .3$

$$\underbrace{P(X \le 10)}_{\text{TABLE!}} = P(X=0) + P(X=1) + \ldots + P(X=10)$$

$\approx$ .048 (TABLE 1)

(RECALL $E[X] = \mu = np = (20)(.7) = 14$)

QUESTION: YOU FLIP A COIN 500 TIMES,

COUNT X = # HEADS. ($n = 500$, $p = .5$, $q = .5$)

FIND $P(X \le 220)$.

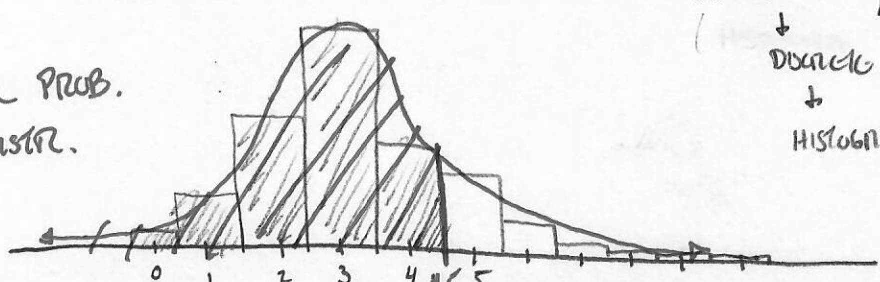
$P(X=0) + P(X=1) + \ldots + P(X=220)$

THERE IS A WAY TO GET A VERY GOOD APPROXIMATION.

IDEA: PROB. DISTRIBUTION FOR X (BINOMIAL) → DISCRETE → HISTOGRAM

SYMM. UNIMODAL

NORMAL PROB. DISTR.



$P(X \le 4) \approx P(X$ … $\le 4.5)$

HW §6.3