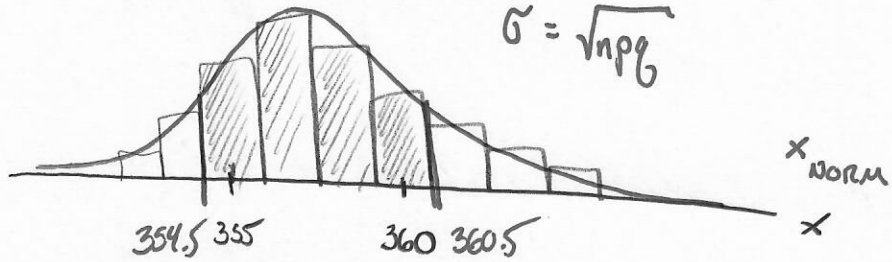
6.44

$\mu = np$

$\sigma = \sqrt{npq}$



$P(355 \le x \le 360)$ x BINOMIAL (DISCRETE)

$= P(x = 355) + P(x = 356) + \dots + P(x = 360)$

RECALL $P(x_{NORM} = 355) = 0$

$= P(354.5 \le x_{NORM} \le 355.5) + P(355.5 \le x_{NORM} \le 356.5)$

$+ \dots + P(359.5 \le x \le 360.5)$

$= P(354.5 \le x_{NORM} \le 360.5)$

$\mu = np = (400)(.9) = 360$

$\sigma = \sqrt{npq}\ \sqrt{(400)(.9)(.1)} = 6$

$z = \dfrac{x - \mu}{\sigma}$

$z = \dfrac{354.5 - 360}{6} = -.9167 \approx -.92$

$z = \dfrac{360.5 - 360}{6} = .0833 \approx .08$

$= P(-.92 \le z \le .08)$

$= P(z \le .08) - P(z \le -.92)$

$.5319 \ -$

10 DIGIT # : $\underset{2-9}{\underline{8}} \times \underbrace{\underline{10} \times \underline{10}}_{0-9} \times \underset{2-9}{\underline{8}} \times \underbrace{\underline{10} \times \underline{10} \times \underline{10} \times \underline{10} \times \underline{10} \times \underline{10}}_{0-9}$

HOW 10 - DIGIT PHONE #'S ? (10 - STAGE EVENT)

$= 8^2 \times 10^8 = \boxed{6,400,000,000}$

0-9 , a-z , A-Z , !@# $

80 CHACTERS

10 - CHARACTER PASSWORD : $\underline{80} \cdot \underline{80} \cdot \underline{80} \cdot$ _ _ _ _ _ _ _

$80^{10} = 1 \times 10^{19}$

1 10,000,000,000,000,000,000

100000000000000 SEC

LICENSE PLATES

K-STAGE EVENTS.

(25)

$\underbrace{\underline{26}\ \underline{26}}_{\text{LETTERS}}\ \underbrace{\underline{10}\ \underline{10}\ \underline{10}\ \underline{10}}_{\text{NUMBERS}}$

PP 2810

PZ 3714

6.76 MILLION

6.30 (d) $P(x \geq 6) = 1 - P(x \leq 5)$

0 1 2 3 4 5

$P(x=6) + P(x=7) + \ldots + P(x=15)$ $\left(C^n_k p^k q^k \right)$

$n = 15$ $p = .5$ $P(x \leq 5) = .151$

$P(x \geq 6) = 1 - .151 = \boxed{.849}$ EXACT

$P(x > 6) = 1 - P(x \leq 6) = 1 - .304 = \boxed{.696}$

APPROX: $P(x \geq 6) = P(x_{norm} \geq 5.5)$

$\mu = np = (15)(.5) = 7.5$

$\sigma = \sqrt{npq} = \sqrt{(15)(.5)(.5)} = 1.9365$

$= P\left(z \geq \dfrac{5.5 - 7.5}{1.9365}\right) = P(z \geq -1.03)$

$= 1 - P(z \leq -1.03)$

$= 1 - .1515$

$= \boxed{.8485} \approx \boxed{.849}$

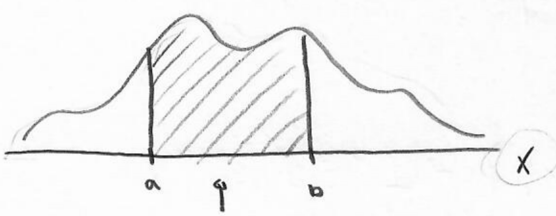
APPROX EXACT

§ 7.4 THE CENTRAL LIMIT THEOREM

DISTRIBUTIONS

OF INDIVIDUAL MEASUREMENTS

TAKEN FROM A POPULATION



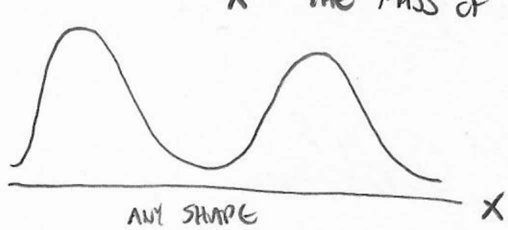
$P(a \leq x \leq b)$

PROB. THAT A SINGLE MEASUREMENT X IS BETWEEN a & b.

DISTRIBUTION COULD HAVE ANY SHAPE!

e.g. EXPERIMENT: SELECT 1 APPLE AT RANDOM FROM A POPULATION OF APPLES, AND RECORD

X = THE MASS OF APPLE.



SAMPLING DISTRIBUTIONS

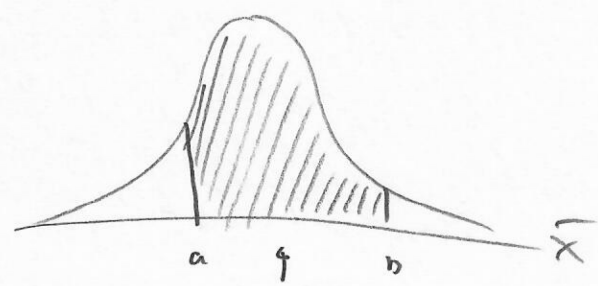
OF AVERAGES (OR SUMS)

OF n MEASUREMENTS TAKEN FROM A POPULATION

SINGLE EXPERIMENT / MEASUREMENT $\overline{X}$

SAMPLE AVERAGE (MEAN)

DISTRIBUTION FOR SAMPLE MEANS $\overline{X}$ IS APPROXIMATELY NORMAL



$P(a \leq \bar{x} \leq b)$

PROB. THAT A SAMPLE OF SIZE n HAS A SAMPLE MEAN $\overline{X}$ BETWEEN a & b.

e.g. EXPERIMENT: SELECT n APPLES AT RANDOM FROM A POPULATION OF APPLES AND RECORD

$\overline{X}$ = THE MEAN MASS OF

CENTRAL LIMIT THEOREM

IF RANDOM SAMPLES OF n MEASUREMENTS ARE DRAWN FROM A POPULATION WITH MEAN μ & STAND. DEV. σ, THEN FOR n LARGE, THE (SAMPLING) DISTRIBUTION OF THE SAMPLE MEAN $\bar{X}$ IS APPROXIMATELY NORMALLY DISTRIBUTED WITH MEAN μ AND STAND. DEV. $\underbrace{\dfrac{\sigma}{\sqrt{n}}}_{\text{"STANDARD ERROR" S.E.}}$

SAME AS POPULATION

THIS HOLD WHEN EITHER

(1) THE POPULATION HAS NORMAL DISTRIBUTION, OR

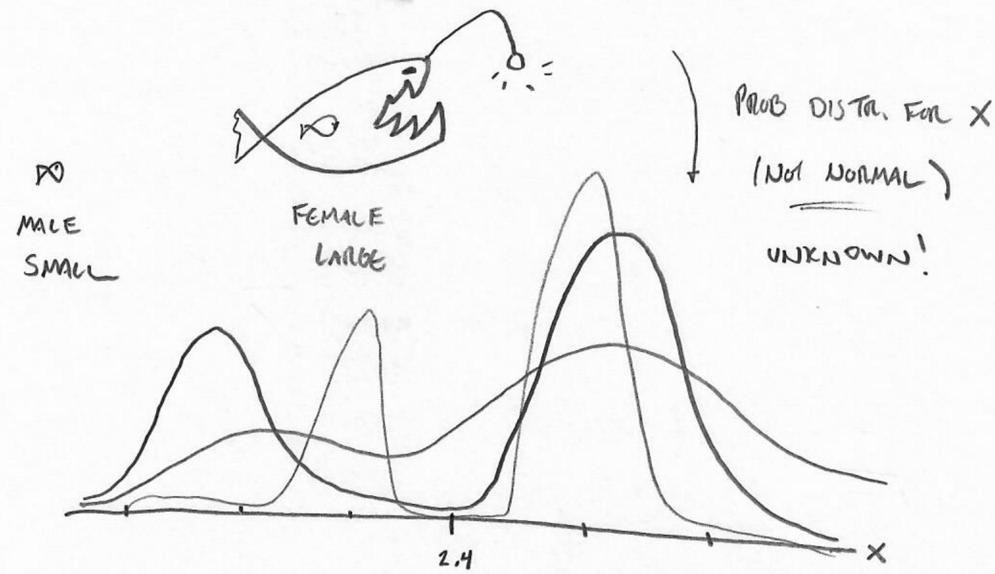
(2) $n \geq 30$.

(ASSUME POPULATION IS VERY LARGE)

CONTINUOUS

JUST 1

ex. LET X = WEIGHT OF A RANDOMLY SELECTED ANGLER FISH



SUPPOSE: POPULATION MEAN $\mu = 2.4$ kg

POP. STAND. DEV. $\sigma = .6$ kg

FIND $P(2.5 \leq X \leq 3)$? CANNOT.

WEIGHT OF A SINGLE FISH.

DISTRIBUTION UNKNOWN

INSTEAD, LETS SAY $\bar{X}$ = MEAN WEIGHT OF A RANDOM SAMPLE OF 50 ANGLER FISH.

CLT => $\bar{X}$ HAS AN APPROXIMATELY <u>NORMAL DISTRIBUTION</u>

(1) MEAN = $\mu = 2.4$ kg

SAME AS POP.

STANDARD ERROR

(2) S.E. = $\frac{\sigma}{\sqrt{n}} = \frac{.6}{\sqrt{50}}$

(1) AVERAGE OF ALL FISH'S WEIGHTS $\longrightarrow$ MEAN FOR X μ (POPULATION MEAN)

AVERAGE TOGETHER ALL SAMPLE MEANS FOR ALL POSSIBLE SAMPLES OF SIZE n $\longrightarrow$ MEAN FOR $\overline{X}$ $= \mu$

MEAN OF SAMPLE MEANS = POPULATION MEAN.

(2) STANDARD ERROR = STANDARD DEVIATION FOR SAMPLE MEANS $\overline{X}$

SAMPLE MEANS HAVE LESS VARIATION THAN INDIVIDUAL MEASUREMENTS

VARIATION FOR SAMPLE MEANS, MEASURED BY STANDARD ERROR, IS

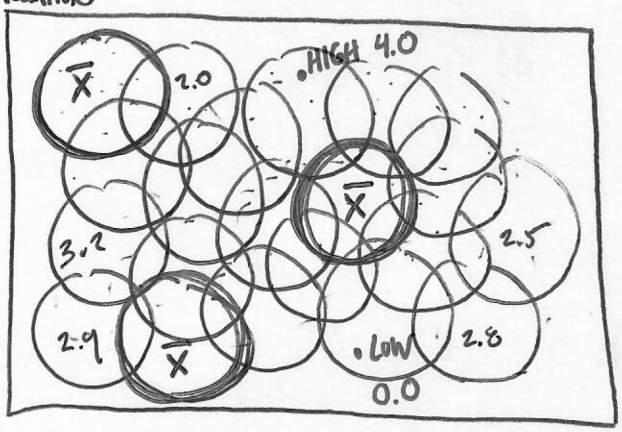
$$S.E. = \frac{\sigma}{\sqrt{n}}$$ (LARGER SAMPLES $\Rightarrow$ LESS VARIATION)

ex. POPULATION : ALL CCNY STUDENTS

X = G.P.A. OF RANDOMLY SELECTED STUDENT

X HAS MEAN μ & STANDARD DEV. σ.

POPULATION



. STUDENTS

○ $\underbrace{\text{CLASSES OF SIZE } n = 30}$

RANDOM SAMPLES OF SIZE 30 (ASSUMPTION)

IF I AVERAGE (THE AVERAGE GPA OF EACH CLASS ($\bar{X}$'s))

WE GET THE AVERAGE GPA OF ALL STUDENTS μ.

$$\text{AVERAGE OF } \underbrace{\text{CLASS GPA's}}_{\bar{X}} = \text{AVERAGE OF } \underbrace{\text{STUDENT GPA's}}_{X}$$

MEAN FOR $\bar{X}$ = μ (MEAN FOR POPULATION)

$$\underline{\text{STANDARD DEVIATION FOR } \bar{X}} = \text{STANDARD ERROR} = \frac{\sigma}{\sqrt{n}} = \frac{\sigma}{\sqrt{30}}$$

(LESS VARIATION IS CLASSROOM GPA's THAN IN STUDENT GPA's)