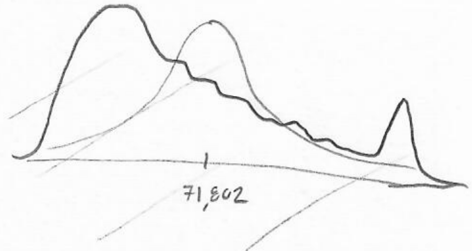
7.26 POPULATION OF PROFESSORS AT 2-YR INSTITUTIONS.



↳ $\underline{\mu = 71{,}802}$, $\underline{\sigma = 4000}$

SAMPLE SIZE $n = 60.$ ← ≥ 30 SO, C.L.T. SAYS

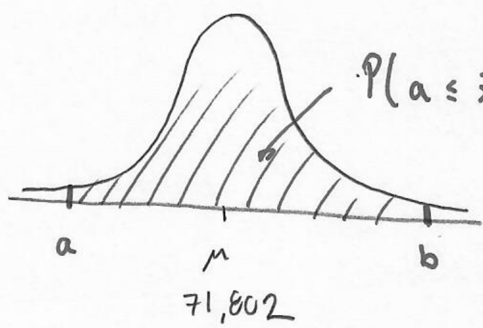
SAMPLE MEAN $\bar{X}$ HAS APPROX. NORMAL DISTR. WITH

(a)

MEAN FOR $\bar{X} = \mu = 71{,}802$

STAND. DEV. FOR $\bar{X}$ = S.E. $= \dfrac{\sigma}{\sqrt{n}} = \dfrac{4000}{\sqrt{60}} = \underline{516.4}$
(STANDARD ERROR)

(b)



$P(a \leq \bar{x} \leq b) = .95$

$\bar{x}$ (SAMPLE MEAN)

EMPERICAL RULE: (R.V. X SYMMETRIC UNIMODAL DISTR.)

APPLY TO $\bar{X}$

$$P(\mu - \sigma \leq X \leq \mu + \sigma) \approx .68$$
$$P(\mu - 2\sigma \leq X \leq \mu + 2\sigma) \approx .95$$
$$P(\mu - 3\sigma \leq X \leq \mu + 3\sigma) \approx .997$$

EMPERICAL RULE: $P(\mu - 2\,\text{S.E.} \leq \bar{X} \leq \mu + 2\,\text{S.E.}) \approx .95$

$71{,}802 - 2(516.40)$ $71{,}802 + 2(516.40)$

$\underline{70{,}769.20}$ $\underline{72{,}834.80}$

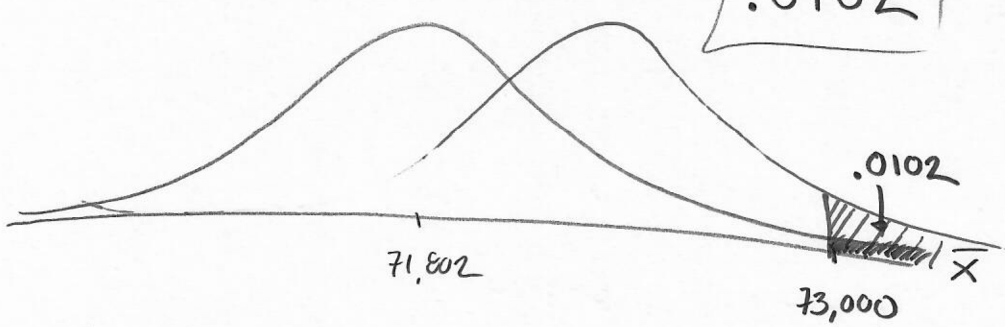
(b) $P(\bar{x} \geq 73{,}000)$ ASSUME $\bar{X}$ IS CONTINUOUS

$$z = \frac{x - \mu}{\sigma} = \frac{\text{RAN. VAR} - \text{MEAN}}{\text{STAND. DEV.}}$$

FOR SAMPLE MEANS: $z = \frac{\bar{x} - \mu}{\text{S.E.}} = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$

$$z = \boxed{\frac{73{,}000 - 71{,}802}{4000 / \sqrt{60}}} = \underline{\underline{2.32}}$$

$$P(\bar{x} \geq 73{,}000) = P(z \geq 2.32)$$
$$= 1 - P(z \leq 2.32)$$
$$= 1 - .9898 = \boxed{.0102}$$



ex. POPULATION OF ALZHEIMERS PATIENTS.

$X =$ TIME FROM ONSET ... TO DEATH.

$\mu = 8$, $\sigma = 4$

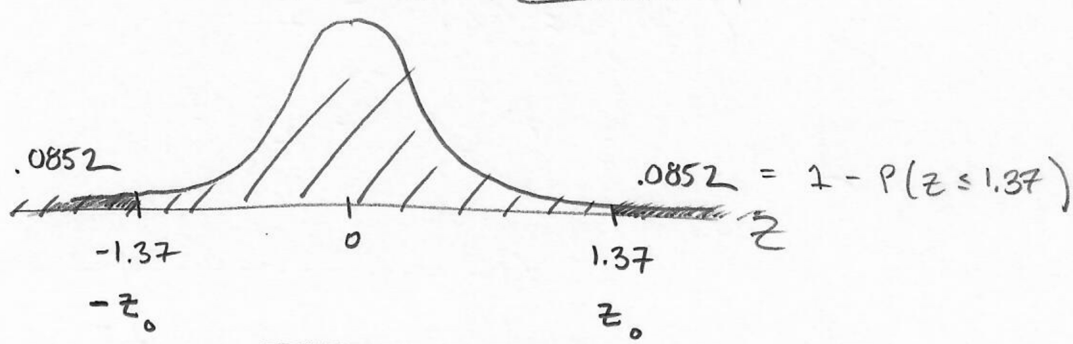
SAMPLE SIZE $n = 30$: $\bar{X}$ HAS APPROX NORMAL DISTR. (CLT)

WITH MEAN $\mu = 8$, S.E. $= \frac{\sigma}{\sqrt{n}} = \frac{4}{\sqrt{30}}$

1. $P(\bar{X} < 7)$

$$z = \frac{\bar{x} - \mu}{S.E.} = \frac{7-8}{4/\sqrt{30}} = -1.37$$

$$= P(z < -1.37) = \cancel{.9147} = \boxed{.0853}$$

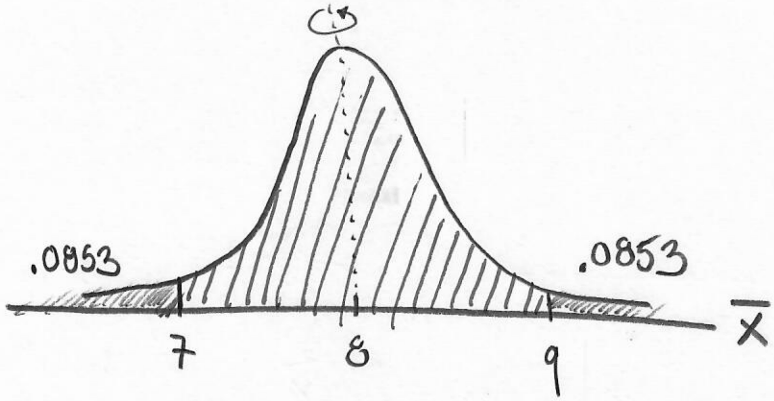


$$\boxed{\begin{aligned} P(z \geq z_0) &= P(z \leq -z_0) \\ &= 1 - P(z \leq z_0) \end{aligned}}$$

2. $P(\bar{X} > 7) = 1 - P(x < 7) = 1 - .0853$

$$= \boxed{.9147}$$

3.



$P(7 < \bar{x} < 9) = 1 - .0853 - .0853$

$= \boxed{.8294}$

CENTRAL LIMIT THM FOR SAMPLE SUMS / TOTALS

SAMPLE MEAN $\bar{X} = \dfrac{\sum x_i}{n}$ IS NORMALLY DISTRIBUTED WITH MEAN μ & S.E. $= \dfrac{\sigma}{\sqrt{n}}$

MULTIPLY BY n

SAMPLE SUM $\sum x_i = n\bar{X}$ IS NORMALLY DISTRIBUTED WITH MEAN $n\mu$ &

RANDOM VAR.

S.E. $= n\dfrac{\sigma}{\sqrt{n}} = \sqrt{n}\cancel{\sqrt{n}}\dfrac{\sigma}{\cancel{\sqrt{n}}} = \sqrt{n}\,\sigma$

NORMALLY DISTRIBUTED $\Rightarrow$ C.L.T. APPLIES REGARDLESS OF SAMPLE SIZE n

7.31 POPULATION OF BANANAS, X = POTASSIUM IN 1 BANANA

MEAN $\mu = 422$, $\sigma = 13$

T IS A SAMPLE SUM/TOTAL WITH $n = 3$.

& CLT $\Rightarrow$ T IS NORMALLY DISTR. WITH

(a) MEAN $n\mu = (3)(422) = 1266$

S.E. $\sqrt{n}\,\sigma = \sqrt{3}(13) = 22.5167$ STANDARD ERROR

(b) $P(T > 1300)$

$$z = \frac{\text{R.V.} - \text{MEAN}}{\text{S.E.}} = \frac{T - n\mu}{\sqrt{n}\,\sigma}$$

$$z = \frac{1300 - (3)(422)}{\sqrt{3}(13)} = 1.51$$

$P(z > 1.51) = 1 - P(z < 1.51) = P(z < -1.51)$

$= \boxed{.0655}$