IF THIS IS TRUE

POPULATION PROPORTION $p = .56$, $q = .44$

SAMPLE SIZE 1000. SAMPLE PROPORTION $\hat{p} = .7$. HOW LIKELY IS THIS?

HIGHER THAN EXPECTED

MEAN (EXPECTED VALUE) FOR $\hat{p} = p = .56$

HOW LIKELY IS IT TO OBTAIN A RANDOM SAMPLE OF SIZE 1000 WITH SAMPLE PROPORTION $\underbrace{\hat{p} \geq .7}_{\text{AS HIGH AS OBSERVED.}}$

$$P(\hat{p} \geq .7) = P\left(z \geq \frac{.7 - .56}{\sqrt{\frac{(.56)(.44)}{1000}}}\right)$$

$$z = \frac{\text{R.V. - MEAN}}{\text{S.E.}} = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}}$$

3.49

$$= P(z \geq 8.92)$$

$$= 1 - P(z \leq 8.92) \approx 1 - 1 = 0$$

IF $p = .56$, WE WOULDN'T EXPECT TO SEE SUCH A LARGE SAMPLE PROPORTION $\hat{p}$. IF WE DID, IT WOULD BE STRONG EVIDENCE THAT p IS ACTUALLY HIGHER THAN .56.

§ 8.4 POINT ESTIMATION & MARGIN OF ERROR.

RECALL C.L.T. SAMPLE MEANS $\bar{X}$ THAT ARE EITHER

- OF SIZE $n \geq 30$, OR
- TAKEN FROM A POPULATION WITH NORMALLY DISTRIBUTED MEASUREMENTS X

ARE APPROXIMATELY NORMALLY DISTRIBUTED WITH MEAN μ & $S.E. = \frac{\sigma}{\sqrt{n}}$.

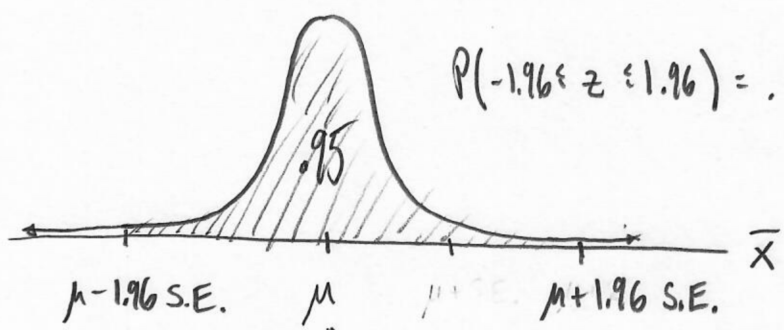
OBTAIN SAMPLE OF SIZE n WITH MEAN $\bar{X}$, AND STAND. DEV. S.

NOW WE WANT TO ESTIMATE THE POPULATION MEAN μ.

(EASY: $\mu \approx \bar{X}$)

PROBABLY NOT ACCURATE.

$P(-1.96 \leq z \leq 1.96) = .95$



UNKNOWN UNKNOWN

$$S.E. = \frac{\sigma}{\sqrt{n}} \approx \frac{s}{\sqrt{n}}$$

THERE IS 95% CHANCE THAT $\bar{X}$ IS WITHIN 1.96 S.E. OF μ.
σ

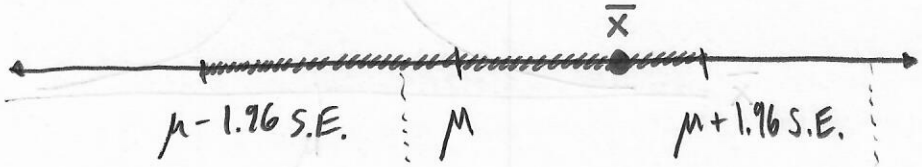
μ . . . . $\bar{X}$.

$$\underbrace{P\left(\bar{x} - 1.96 \text{ S.E.} \le \mu \le \bar{x} + 1.96 \text{ S.E.}\right)}_{} = .95$$

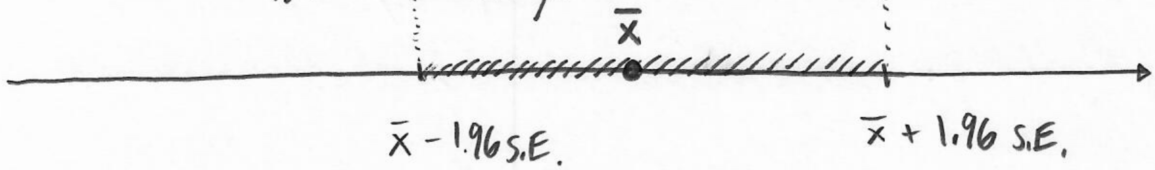
$$\mu \approx \bar{X} \pm 1.96 \text{ S.E.}$$

THERE IS A 95% CHANCE THAT μ IS IN THIS INTERVAL.

C.L.T. $\Rightarrow$ 95% CHANCE THAT $\bar{X}$ IS WITHIN 1.96 S.E. OF μ.



95% CHANCE THAT μ IS WITHIN 1.96. S.E. OF $\bar{X}$



NOTE: $\text{S.E.} = \dfrac{\sigma}{\sqrt{n}}$ ← UNKNOWN POPULATION PARAMETER.

$\therefore \text{S.E.} \approx \dfrac{s}{\sqrt{n}}$ ← SAMPLE STAND. DEV. KNOWN.

$$\left(\sigma = \frac{\sum (x-\mu)^2}{n} \qquad s = \frac{\sum (x-\bar{x})^2}{n-1} \right)$$

IN SUMMARY,

POPULATION MEAN $\mu \approx \bar{X}$.

HAS S.E. $= \frac{\sigma}{\sqrt{n}} \approx \frac{s}{\sqrt{n}}$

MARGIN OF ERROR $= 1.96$ S.E. $\approx 1.96 \frac{s}{\sqrt{n}}$

$\hookrightarrow \therefore \mu \approx \bar{X}$ WITH MARGIN OF ERROR $1.96 \frac{s}{\sqrt{n}}$

$\therefore \mu \approx \bar{X} \pm 1.96 \frac{s}{\sqrt{n}}$

POPULATION PROPORTION $p \approx \hat{p}$

HAS S.E. $= \sqrt{\frac{pq}{n}} \approx \sqrt{\frac{\hat{p}\hat{q}}{n}}$

MARGIN OF ERROR $= 1.96$ S.E. $\approx 1.96\sqrt{\frac{\hat{p}\hat{q}}{n}}$

$\hookrightarrow \therefore p \approx \hat{p}$ WITH MARGIN OF ERROR $1.96\sqrt{\frac{\hat{p}\hat{q}}{n}}$

$\therefore p \approx \hat{p} \pm 1.96\sqrt{\frac{\hat{p}\hat{q}}{n}}$

8.12 POP MEAN $\approx$ SAMPLE MEAN (POINT ESTIMATE)

$$\mu \approx \bar{X}$$

$\bar{X}$ HAS NORMAL DIST'N.

$$\mu \approx 56.4$$

$$\text{MARGIN OF ERROR} = 1.96 \frac{\sigma}{\sqrt{n}} \approx 1.96 \frac{s}{\sqrt{n}} = 1.96 \frac{\sqrt{2.6}}{\sqrt{50}}$$

$$\left(s = \sqrt{s^2} = \sqrt{2.6}\ ,\ n = 50 \right)$$

$$\text{MARGIN OF ERROR} = .4469$$

i.e. $\mu \approx 56.4 \pm .4469$

95% CHANCE THAT μ IS IN THIS INTERVAL.

$$56.4 - .4469 \leq \mu \leq 56.4 + .4469$$

$$P\left(55.9531 \leq \mu \leq 56.8469\right) = .95$$

Def: $\boxed{\text{MARGIN OF ERROR} = 1.96 \text{ S.E.}}$

ex. A SAMPLE OF SIZE 225 MEASUREMENTS ARE OBTAINED WITH

$\bar{x} = 82.3$, $s = 5.6$.

0.3092

ESTIMATE THE POPULATION MEAN μ, AND FIND THE MARGIN OF ERROR.

(a) $\mu \approx \bar{x} = \underline{\underline{82.3}}$

$\bar{x}$ SAFE ESTIMATE

(b) $\boxed{\text{MARGIN OF ERROR} = 1.96\,(\text{S.E.})} = 1.96\,\frac{\sigma}{\sqrt{n}} \approx 1.96\,\frac{s}{\sqrt{n}}$

HOW GOOD IS THE ESTIMATE?

$= 1.96\,\frac{5.6}{\sqrt{225}} = .7317$

8.13. SAMPLE: SIZE $n = 500$

$x = \#\text{ SUCCESSES} = 450$

SAMPLE PROPORTION $\hat{p} = \frac{x}{n} = \frac{450}{500} = .9$

$\hat{q} = 1 - \hat{p} = .1$

$\hat{p}$ (NOT $\bar{x}$)

POPULATION PROPORTION $p \approx \hat{p} = .9$

$\boxed{\text{MARGIN OF ERROR} = 1.96\,(\text{S.E.})} = 1.96\sqrt{\frac{pq}{n}} \approx 1.96\sqrt{\frac{\hat{p}\hat{q}}{n}}$

$= 1.96\sqrt{\frac{(.9)(.1)}{500}} = .0263$

$p \approx .9 \pm .0263$

90% ± 2.63%

Note: • When estimating μ, margin of error is

$$1.96 \text{ S.E.} \approx 1.96 \frac{s}{\sqrt{n}}$$

• When estimating p, margin of error is

$$1.96 \text{ S.E.} \approx 1.96 \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

In both cases, margin of error decreases as n increases.

Larger samples cause smaller margins of error, i.e. higher accuracy.

Note: The standard error for sample proportion $\hat{p}$

$$\text{S.E.} = \sqrt{\frac{pq}{n}}$$, for fixed sample size n,

is maximized when $\underbrace{p = q = .5}_{\text{most uncertainty}}$.

4. INTERPRETATION: 95% CHANCE THAT POP. PROP. p

IS BETWEEN $\underbrace{.9 - .0263 \quad \& \quad .9 + .0263}$

$.8737 \quad \& \quad .9263$

$(87.37\% \quad \& \quad 92.63\%)$

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ex. SUPPOSE A RANDOM SAMPLE OF $n = 1500$ VOTERS IS PULLED AND ASKED IF THEY WILL VOTE FOR CANDIDATE X. 823 SAY YES, 677 SAY NO.

ESTIMATE THE PROPORTION OF ALL VOTERS THAT WILL VOTE FOR CANDIDATE X (INCLUDE A MARGIN OF ERROR).

SAMPLE: $n = 1500$, $x = 823$ = # SUCCESSES IN SAMPLE

$$\hat{p} = \frac{x}{n} = \frac{823}{1500} = .5487$$

.0251

$$\hat{q} = 1 - \hat{p} = .4513$$

$p \approx \hat{p} = .5487$, MARGIN OF ERROR 1.96 S.E. $= 1.96\sqrt{\frac{pq}{n}}$

$$= 1.96\sqrt{\frac{\hat{p}\hat{q}}{n}} = 1.96\sqrt{\frac{(.5487)(.4513)}{1500}}$$

$54.87 \pm 2.52\%$

$= .0252$

$55\% \pm 2.5\%$

$52.5 - 57.5\%$

ex. SUPPOSE YOU WANT TO ESTIMATE A POPULATION PROPORTION WITH A SAMPLE PROPORTION & WITH A MARGIN OF ERROR LESS THAN OR EQUAL TO .1% (.001).

HOW LARGE MUST YOUR SAMPLE SIZE BE?

$$\text{MARGIN OF ERROR} \leq .001$$

$$1.96\sqrt{\frac{pq}{n}} \leq .001$$

$$\sqrt{\frac{pq}{n}} \leq \frac{.001}{1.96}$$

ASSUME THE WORST!

$p = q = .5$

$$\sqrt{\frac{(.5)(.5)}{n}} \leq \frac{.001}{1.96}$$

SOLVE THE INEQUALITY FOR n

$$\left(\sqrt{\frac{.25}{n}}\right)^2 \leq \left(\frac{.001}{1.96}\right)^2 \rightarrow \frac{.25}{n} \leq \left(\frac{.001}{1.96}\right)^2$$

$$.25 \leq n\left(\frac{.001}{1.96}\right)^2 \rightarrow \frac{.25}{\left(\frac{.001}{1.96}\right)^2} \leq n$$

$$960{,}400 \le n$$

USE $p = .5487$

$q = .4513$

$$1.96\sqrt{\frac{(.5487)(.4513)}{n}} \le .001$$

$$951{,}288.9 = \frac{(.5487)(.4513)}{\left(\frac{.001}{1.96}\right)^2} \le n$$

SAMPLE SIZE $n \ge 951{,}289$ (ROUND UP ALWAYS!)

HW §8.4