8.19

(a) POPULATION PROPORTION $p \approx \hat{p} = .75$

$$\text{MARGIN OF ERROR} = 1.96 \text{ S.E.} = 1.96\sqrt{\frac{pq}{n}} \approx 1.96\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$1.96\sqrt{\frac{(.75)(.25)}{1004}} = .0268 \rightarrow 2.68\%$$

(b) MAXIMUM $\underbrace{\text{MARGIN OF ERROR}}_{1.96 \text{ S.E.}}$ OCCURS WHEN $\underbrace{p = \hat{q} = .5}_{\text{WORST CASE!}}$ (HIGHEST UNCERTAINTY)

$$1.96\sqrt{\frac{(.5)(.5)}{1004}} = .0309 \rightarrow \underbrace{3.09\%}_{\text{MAX M.O.E.}}$$

M.O.E $\pm$ 3.5% INCORRECT.

§ 8.5 INTERVAL ESTIMATION (CONFIDENCE INTERVALS)

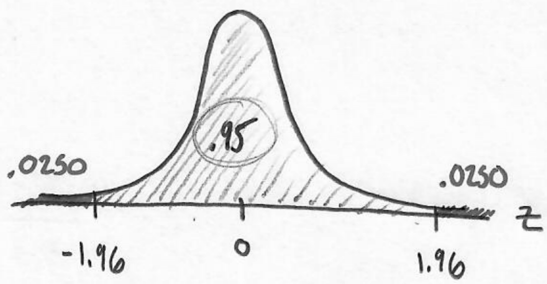
95% CONFIDENCE INTERVAL

SO FAR: POP. MEAN $\mu \approx \bar{x} \pm \boxed{1.96 \text{ S.E.}} \approx \bar{x} \pm 1.96 \frac{s}{\sqrt{n}}$

"MARGIN OF ERROR"

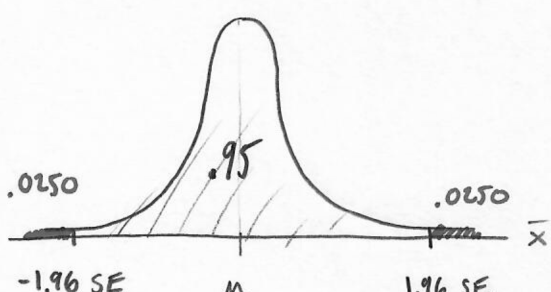
POP PROP $p \approx \hat{p} \pm \boxed{1.96 \text{ S.E.}} \approx \hat{p} \pm 1.96 \sqrt{\frac{\hat{p}\hat{q}}{n}}$

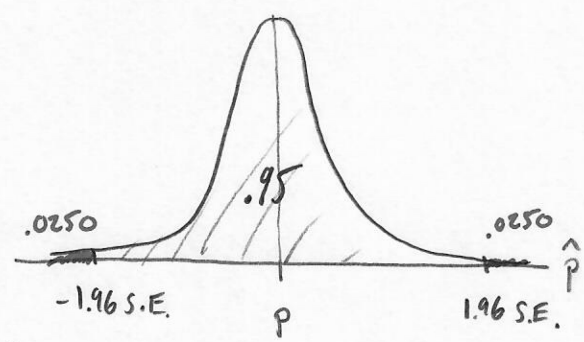
WHY 1.96? →



$P(-1.96 \leq z \leq 1.96) = .95$

C.L.T. $\bar{x}$ & $\hat{p}$ ARE NORMALLY DISTRIBUTED WITH MEAN μ & p, RESPECTIVELY (UNBIASED).





PROB. .95 THAT $\bar{x}$ LIES WITHIN 1.96 S.E OF μ

PROB. .95 THAT μ LIES WITHIN 1.96 S.E. OF $\bar{x}$

TO 95% SURE, WE USE 1.96

PROB. .95 THAT $\hat{p}$ LIES WITHIN 1.96 S.E. OF p

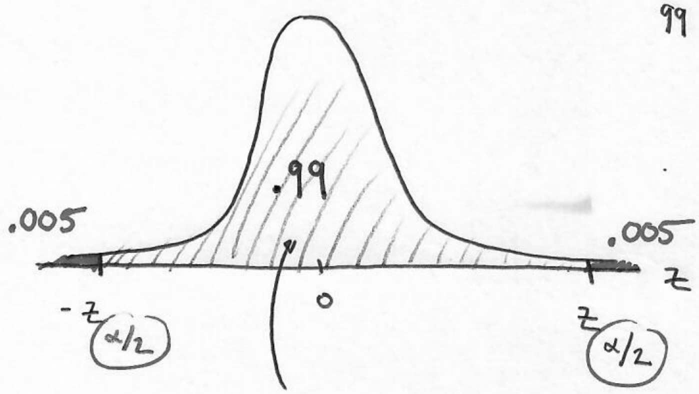
PROB. .95 THAT p LIES WITHIN 1.96 S.E. OF $\hat{p}$

95% CONFIDENCE INTERVAL

95% CONFIDENT THAT μ LIES IN THE INTERVAL $[\bar{x} - 1.96 \text{ S.E.}, \bar{x} + 1.96 \text{ S.E.}]$

99% CONFIDENCE INTERVAL

Note:



FIND $z_{\alpha/2}$

PROB. .99

$\alpha = 1 - .99 = .01$ IS SPLIT BETWEEN THE TAILS.

i.e. EACH TAIL CONTAINS PROB. $\frac{\alpha}{2} = \frac{.01}{2} = .005$

$P(z \le -z_{\alpha/2}) = .0050 \Rightarrow -z_{\alpha/2} = -2.58$ (ROUND UP TO BE SAFE)

$z_{\alpha/2} = 2.58$

$P(-2.58 \le z \le 2.58) = .99$

CLT. $\Rightarrow P(\mu - 2.58\,S.E \le \bar{X} \le \mu + 2.58\,S.E) = .99$

99% SURE THAT $\bar{X}$ LIES WITHIN 2.58 S.E. OF μ

99% SURE THAT μ LIES WITHIN 2.58 S.E. OF $\bar{X}$.

99% CONFIDENCE INTERVAL

$[\bar{X} - 2.58\,S.E.\ ,\ \bar{X} + 2.58\,S.E.]$

$\bar{X} \pm \boxed{2.58}^{*}\,S.E.$

$\mu \approx \bar{X} \pm 1.96$ S.E.

95% CONF. INT.

SMALLER INTERVAL

=> LESS CONFIDENCE

=> MORE ACCURACY.
(PRECISION)

$\mu \approx \bar{X} \pm 2.58$ S.E.

99% CONF. INT.

LARGER INTERVAL

=> HIGHER CONFIDENCE

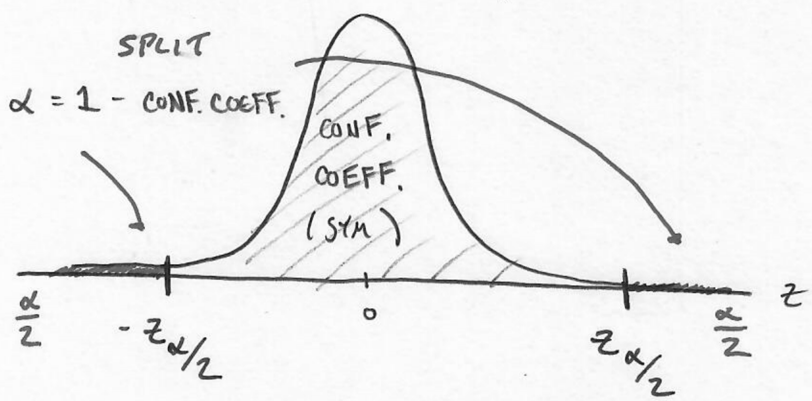
=> LESS ACCURACY
(PRECISION)

e.g.

	ESTIMATE HEIGHT OF MT. EVEREST	CONFIDENCE	
LESS PRECISE	5,000 ft - 15,000 ft	.9	← MORE CONF.
	10,000 ft - 12,000 ft	.3	
	PRECISE	NOT CONFIDENT	
	0 ft - 1,000,000 ft	.9999	
	25,888 - 25,889	.0001	

	CONFIDENCE COEFFICIENT	(1 - CONF. COEFF.) α	$\frac{\alpha}{2}$	$z_{\alpha/2}$
	.99	.01	.0050	2.58
	.98	.02	.0100	2.33
DEFAULT * "MARGIN OF ERROR"	.95	.05	.0250	1.96
	.9	.1	.0500	1.645

KNOW THESE #'S



ex. SUPPOSE A SAMPLE OF $n = 150$ MEASUREMENTS HAS A SAMPLE MEAN $\bar{x} = 58.4$ & SAMPLE STAND. DEV. $s = 5.2$.

CREATE A 98% CONFIDENCE INTERVAL FOR POP MEAN μ.

$$\mu \approx \bar{x} \pm \underline{2.33}\ S.E. \approx 58.4 \pm 2.33 \frac{5.2}{\sqrt{150}}$$

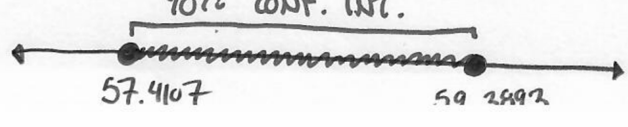
$$S.E. = \frac{\sigma}{\sqrt{n}} \approx \frac{s}{\sqrt{n}}$$

$$\mu \approx \underline{58.4 \pm .9893} \quad (1)$$

(2) $[57.4107, 59.3893]$

98% CONF. INT.

(3)



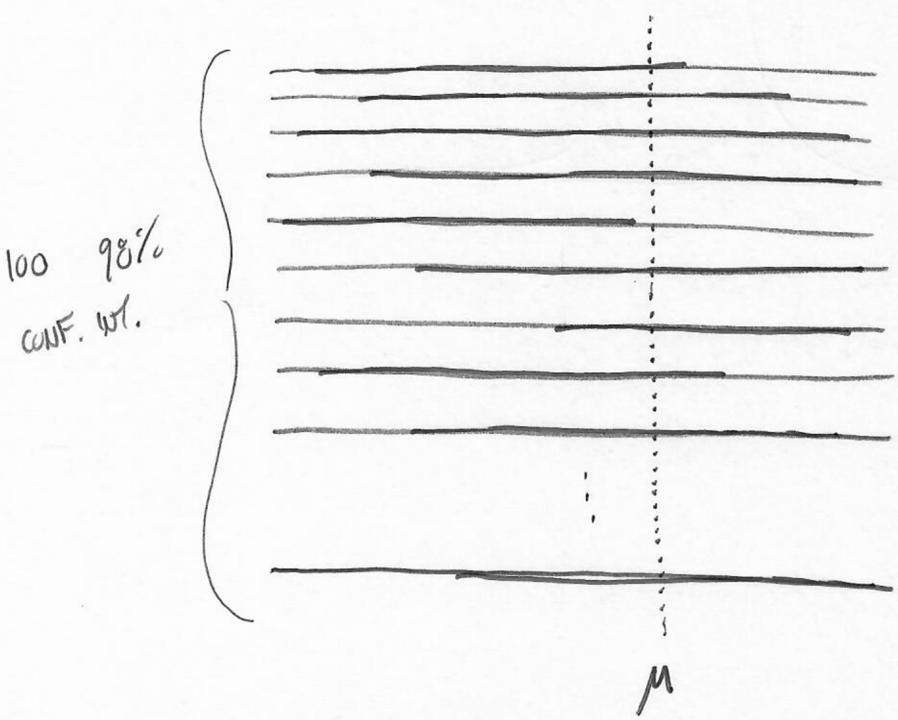
(4) $57.4107 \leq \mu \leq 59.3893$

INTERPRETATION : $P(57.4107 \leq \mu \leq 59.3893) = .98$

IF WE WERE TO REPEAT THIS EXPERIMENT OVER & OVER (COLLECT RANDOM SAMPLE OF SAME SIZE n, COMPUTE $\bar{x}$ & s, GENERATE A 98% CONF. INTERVAL)

DIFFERENT EACH TIME

SO WE GET A BUNCH OF 98% CONFIDENCE INTERVALS.



THE TRUE POPULATION MEAN WILL LIE IN 98% OF THESE INTERVALS.

$P(\text{INT. CONTAINS } \mu) = .98$

$P(2 \text{ INT. CONTAIN } \mu) = (.98)(.98) = .9604$

TO CREATE A CONFIDENCE INTERVAL:

$$\mu \approx \bar{x} \pm z_{\alpha/2} \text{ S.E.} \approx \bar{x} \pm z_{\alpha/2} \frac{s}{\sqrt{n}}$$

$$p \approx \hat{p} \pm z_{\alpha/2} \text{ S.E.} \approx \hat{p} \pm z_{\alpha/2} \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

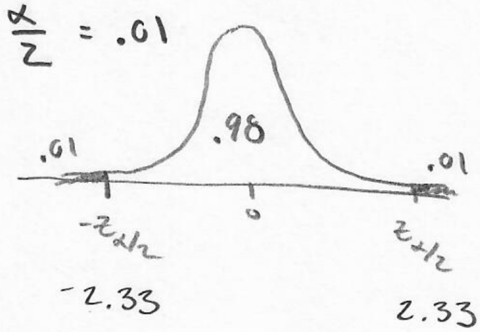
$z_{\alpha/2}$ DETERMINED BY CONFIDENCE COEFF.

8.40 (a) SAMPLE: SIZE $n = 469$

PROPORTION $\hat{p} = .77$, $\hat{q} = .23$

98% CONF COEFF $\Rightarrow \alpha = .02 \Rightarrow \frac{\alpha}{2} = .01$

$z_{\alpha/2} = 2.33$



$$p \approx \hat{p} \pm z_{\alpha/2} \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$p \approx .77 \pm 2.33 \sqrt{\frac{(.77)(.23)}{469}} = .77 \pm .0453$$

No, .85 IS NOT CONTAINED IN THE 98% CONF. INT.

$$P(.7247 \le p \le .8153) = .98$$