DISCRETE

$\downarrow$

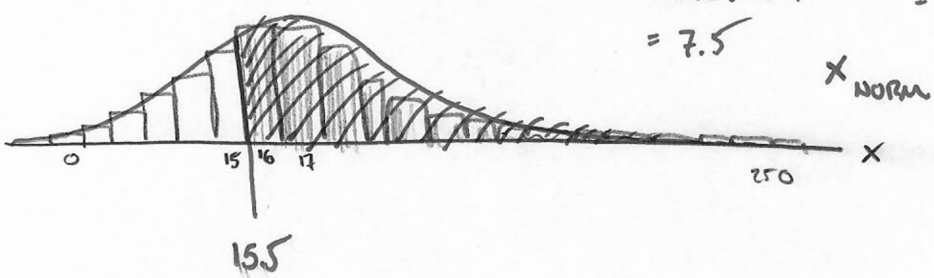
X BINOMIAL , $n = 250$ TRIALS

$p = .03$, $q = .97$

$P(X > 15) = P(X \geq 16)$

$\mu = np$
$= 250(.03)$
$= 7.5$

$\sigma = \sqrt{npq}$
$= \sqrt{(250)(.03)(.97)}$
≈ 2.6972



$\approx P(X_{NORM} \geq 15.5) = P\left(Z \geq \dfrac{15.5 - 7.5}{2.6972}\right)$

$\left(Z = \dfrac{X - \mu}{\sigma}\right)$

$= 1 - P(Z \leq 2.97)$

$= 1 - .9985 = \underline{\underline{.0015}}$

SAMPLE SIZE $n = 256$

$\bar{X}$ NORM. DISTR. μ $\frac{\sigma}{\sqrt{n}}$

SAMPLE TOTAL

$\sum X$ NORM DISTR $n\mu$ $\sqrt{n}\,\sigma$

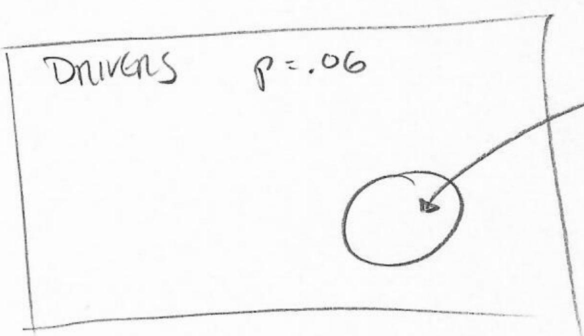
$$P\left(\sum x > 45{,}000\right) = P\left(z \geq \frac{45000 - n\mu}{\sqrt{n}\,\sigma}\right)$$

$$= P\left(z > \frac{45000 - (256)(175)}{\sqrt{256}\,(40)}\right)$$

$$= P(z > .3125) = 1 - P(z \leq \underbrace{.3125}_{.31})$$

$$= 1 - .6217$$

$$= \underline{\underline{.3783}}$$



SAMPLE SIZE $n = 300$

$$P(\hat{p} > .08)$$

$\hat{p}$ NORM. DISTR (C.L.T.)

MEAN $p = .06$

S.E. $\sqrt{\frac{pq}{n}} = \sqrt{\frac{(.06)(.94)}{300}} \approx$

$\approx .0137$

$$P(\hat{p} > .08) = P\left(z > \frac{.08 - .06}{.0137}\right)$$

$$\approx P(z > 1.46) = 1 - P(z \leq 1.46)$$

$$= 1 - .9279 = .0721$$

$$p \approx \hat{p} = \frac{319}{550} = .58 \qquad \left(\hat{p} = \frac{x}{n}\right)$$

MARGIN OF ERROR = 1.96 S.E.

$$\approx 1.96\sqrt{\frac{\hat{p}\hat{q}}{n}} = 1.96\sqrt{\frac{(.58)(.42)}{550}}$$

$$= \underline{\underline{.0412}}$$

$$p \approx .58 \pm .0412$$

(58% ± 4%)

$$\mu \approx \overline{X} \pm z_{\alpha/2}\ \text{S.E.}$$

$$\mu \approx \overline{X} \pm 2.58\frac{s}{\sqrt{n}} = 7.6 \pm 2.58\left(\frac{2.3}{\sqrt{50}}\right)$$

Lower Bound 6.7608

Upper Bound 8.4392

$\bar{x}_1 = 12$ $\quad\quad$ $\bar{x}_2 = 9$

$s_1 = 5$ $\quad\quad$ $s_2 = 2$

$n_1 = 458$ $\quad\quad$ $n_2 = 544$

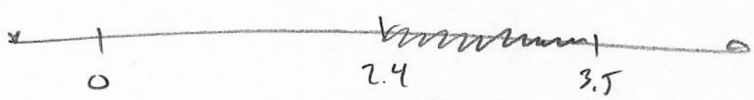
$$(\mu_1 - \mu_2) \approx (\bar{x}_1 - \bar{x}_2) \pm \overbrace{2.33\sqrt{\frac{s_1^2}{458\,n_1} + \frac{s_2^2}{544\,n_2}}}^{z_{\alpha/2} \quad S.E.}$$

$$\approx 12 - 9 \pm 2.33\sqrt{\frac{5^2}{458} + \frac{2^2}{544}}$$

$\approx$ Lower Bound $\quad$ 2.4201

Upper Bound $\quad$ 3.5799

98% $\quad\quad$ Pos.



$\mu_1 - \mu_2$

$$\mu_1 - \mu_2 > 0 \Rightarrow \underline{\underline{\mu_1 > \mu_2}}$$

$$+\mu_2 \quad +\mu_2$$

<u>Yes.</u> (True.)

§8.9 CHOOSING THE SAMPLE SIZE.

$$\text{POPULATION PARAMETER} \approx \text{SAMPLE STATISTIC} \pm \overbrace{z_{\alpha/2}\ \text{S.E.}}^{\text{ERROR}}$$

1. $\mu \approx \bar{x} \pm z_{\alpha/2} \dfrac{s}{\sqrt{n}}$

2. $p \approx \hat{p} \pm z_{\alpha/2} \sqrt{\dfrac{\hat{p}\hat{q}}{n}}$

3. $\mu_1 - \mu_2 \approx \bar{x}_1 - \bar{x}_2 \pm z_{\alpha/2} \sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}$

4. $p_1 - p_2 \approx \hat{p}_1 - \hat{p}_2 \pm z_{\alpha/2} \sqrt{\dfrac{\hat{p}_1\hat{q}_1}{n_1} + \dfrac{\hat{p}_2\hat{q}_2}{n_2}}$

IF YOU WANT YOUR ERROR TO BE $\le B$ ("ERROR BOUND")

THEN WE MUST HAVE

$$\text{ERROR} = \boxed{z_{\alpha/2}\ \text{S.E.} \le B}$$

THE ONLY THING WE CAN CONTROL IS THE SAMPLE SIZE(S) n (n_1, n_2)

1. $z_{\alpha/2}$ S.E. $\leq B$

VALUE FOR WILL BE APPROXIMATED

$$z_{\alpha/2} \frac{s}{\sqrt{n}} \leq B$$

* DEFAULT VALUE FOR $z_{\alpha/2} = 1.96$

$$\left(z_{\alpha/2} \frac{s}{B}\right)^2 \leq n$$

2. $z_{\alpha/2}$ S.E. $\leq B$

APPROXIMATIONS FOR p, q MAY BE PROVIDED.

$$z_{\alpha/2} \sqrt{\frac{pq}{n}} \leq B$$

IF NOT, ASSUME WORST CASE SCENARIO: $p = q = .5$

$$(z_{\alpha/2})^2 \frac{pq}{B^2} \leq n$$

3. $z_{\alpha/2}$ S.E. $\leq B$

GIVEN APPROX. FOR s_1, s_2

$$z_{\alpha/2} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \leq B$$

ASSUME $n_1 = n_2 = n$

$$z_{\alpha/2}\sqrt{\frac{s_1^2+s_2^2}{n}} \le B$$

$$(z_{\alpha/2})^2\,\frac{s_1^2+s_2^2}{B^2} \le n$$

4. $z_{\alpha/2}\ S.E. \le B$

$$z_{\alpha/2}\sqrt{\frac{p_1q_1}{n_1}+\frac{p_2q_2}{n_2}} \le B$$

APPROX. FOR p_1, p_2 MAY BE GIVEN.
IF NOT, ASSUME THE WORST:
$p_1 = p_2 = q_1 = q_2 = .5$

ASSUME $n_1 = n_2 = n$

$$z_{\alpha/2}\sqrt{\frac{.5}{n}} \le B$$

$$(z_{\alpha/2})^2\,\frac{.5}{B^2} \le n$$

(ALWAYS ROUND UP!)

8.79 APPROXIMATE μ. (1)

$$\mu \approx \bar{X} \pm \underbrace{z_{\alpha/2}\ S.E.}$$

$$1.96 \frac{\sigma}{\sqrt{n}} \le 2$$

ERROR

$$1.96 \frac{10}{\sqrt{n}} \le 2$$ SOLVE FOR n

$$1.96 \cdot 10 \le 2\sqrt{n}$$

$$\frac{1.96 \cdot 10}{2} \le \sqrt{n}$$

$$\left(\frac{1.96 \cdot 10}{2}\right)^2 \le n$$

(2.58 99% C.L.)

$$96.04 \le n \quad \rightsquigarrow \quad \boxed{n \ge 97}$$

$\rightsquigarrow$ ~~166~~ 167

e.g. 73% 51%

8.76

Pop 1 (REP) Pop 2 (DEM)

p_1 p_2

$$p_1 - p_2 \approx \hat{p}_1 - \hat{p}_2 \pm z_{\alpha/2}\,S.E.$$

ERROR

ERROR $\leq .03$

ALL PROP. $= .5$

$$1.96\sqrt{\frac{p_1 q_1}{n_1} + \frac{p_2 q_2}{n_2}} = 1.96\sqrt{\frac{(.5)(.5)+(.5)(.5)}{n}}$$

$n_1 = n_2 = n$

$$= 1.96\sqrt{\frac{.5}{n}} \leq .03$$

SOLVE FOR n

$$(1.96)^2\,\frac{.5}{n} \leq (.03)^2$$

$$(1.96)^2(.5) \leq n(.03)^2$$

$n \geq 2135$

$$\frac{(1.96)^2(.5)}{(.03)^2} \leq n \Rightarrow 2134.2 \leq n$$