§9.1 - 9.3 HYPOTHESIS TESTING

e.g. SUPPOSE IT IS CLAIMED THAT THE AVERAGE HOUSEHOLD INCOME IN YOUR NEIGHBORHOOD IS \$56,000 / YR.

YOU DOUBT THIS, AND SUSPECT THAT THE AVERAGE H.H. INCOME IS ACTUALLY HIGHER, AND YOU PERFORM A STATISTICAL EXPERIMENT TO SHOW THAT THIS IS THE CASE.

CLAIM: $\mu = 56{,}000$

WANT TO SHOW: $\mu > 56{,}000$

NULL HYPOTHESIS H_0 : $\mu = 56{,}000$ (ASSUMPTION, OPPOSITE OF H_A)

ALTERNATING HYPOTHESIS H_A : $\mu > 56{,}000$ (WHAT YOU WANT TO SHOW)

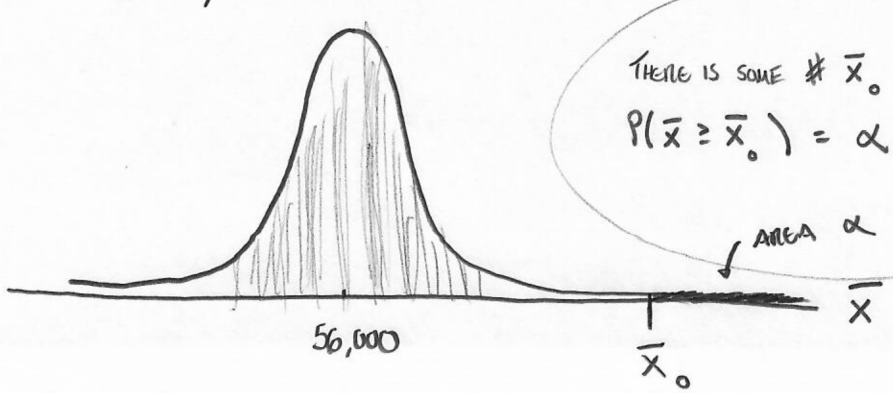
IDEA: GATHER A SAMPLE SIZE n → SAMPLE STATISTICS $\bar{X}$, S

$\bar{X}$: NORM. DISTR. MEAN $= \mu$

S.E. $\approx \frac{S}{\sqrt{n}}$

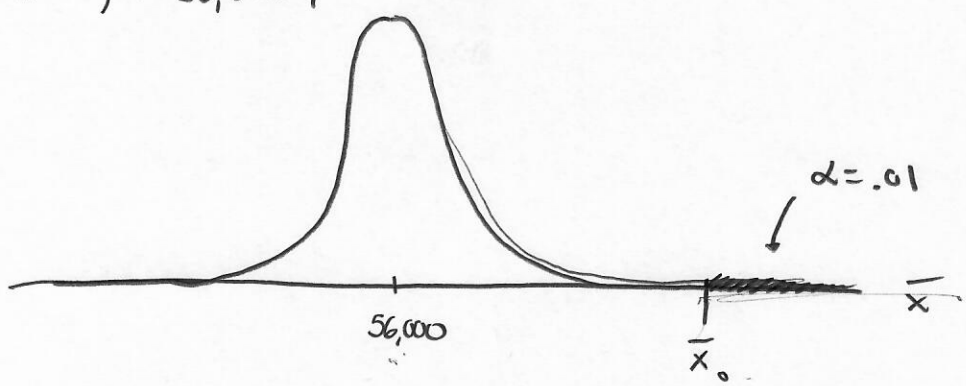
ASSUMING H_0 : $\mu = 56{,}000$

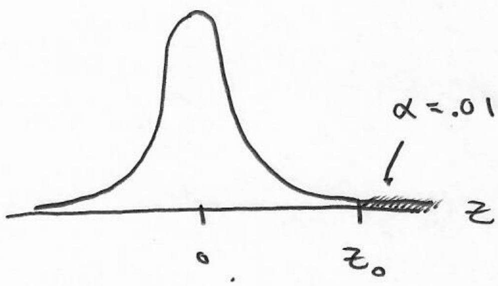
THERE IS SOME # $\bar{x}_0$ SUCH THAT $P(\bar{x} \geq \bar{x}_0) = \alpha$



WHAT IS α? COMON VALUES: .01, .02, .05

WE WOULD CONSIDER IT TO BE STRONG EVIDENCE THAT $\mu > 56{,}000$ IF $\bar{X}$ WAS SO LARGE, THAT SUCH A LARGE VALUE OF $\bar{X}$ WOULD ONLY HAPPEN BY CHANCE WITH PROBABILITY $\leq \alpha$, ASSMING $\mu = 56{,}000$.





$\alpha = .01 \longleftrightarrow P(z \geq z_0) = .01$

$z_0 = 2.33$

$$z_0 = \frac{\bar{x}_0 - \mu}{S.E.}$$

SAY WE KNOW $\sigma = 2{,}500$. THEN $S.E. = \dfrac{\sigma}{\sqrt{n}} = \dfrac{2500}{\sqrt{100}} = 250$

$n = 100$

$$2.33 = \frac{\bar{x}_0 - 56{,}000}{250}$$

$$\bar{x}_0 = 56{,}000 + 2.33(250)$$

$$= 56{,}582.50.$$

ASSUMING $H_0: \mu = 56{,}000$, $P(\bar{X} \geq 56{,}582.5) = .01$.

α

α IS THE VALUE THAT WE PICK IN ADVANCE

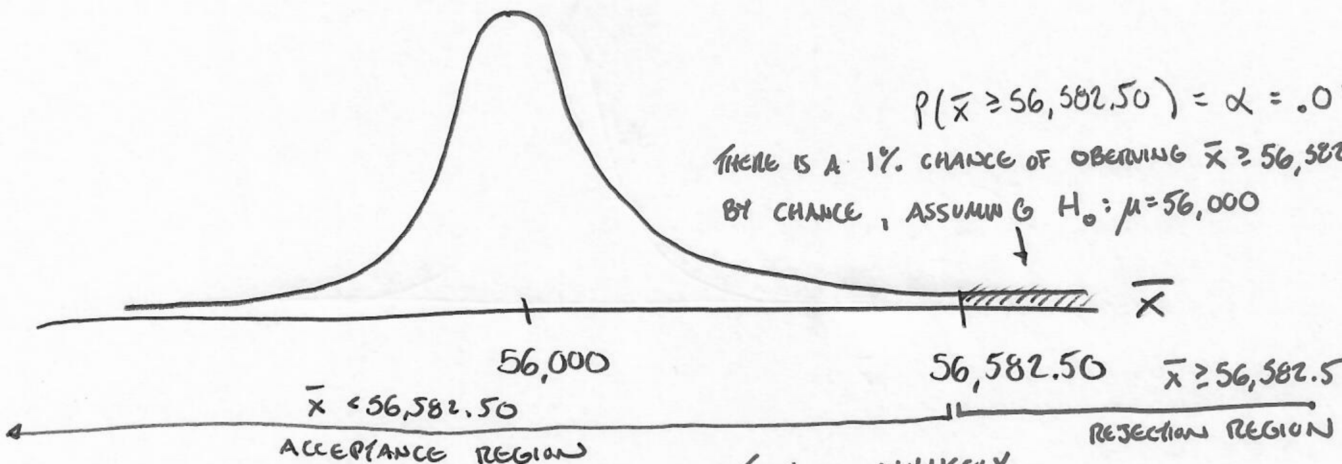
TO BE THE PROBILITY OF A "RARE" EVENT,

i.e "UNLIKELY EVENT".

$P(\bar{x} \geq 56{,}582.50) = \alpha = .01$

THERE IS A 1% CHANCE OF OBSERVING $\bar{x} \geq 56{,}582.5$

BY CHANCE, ASSUMING $H_0: \mu = 56{,}000$



IF $\bar{X} < 56{,}582.5$

THEN WE DO NOT REJECT

THE NULL HYPOTHESIS.

WE ACCEPT THE NULL HYP. H_0

THAT IS UNLIKELY.

SO IF WE OBSERVE $\bar{X} \geq 56{,}582.5$

THEN WE CONSIDER IT TO BE STRONG

EVIDENCE THAT H_0 IS WRONG.

AND WE WOULD REJECT THE NULL HYP. H_0,

IN FAVOR OF ALTERNATIVE HYP. H_A.

ex. SUPPOSE A MANUFACTURER CLAIMS THAT ONLY 6%
OF ITS PRODUCTS HAVE DEFECTS.

YOU WANT TO SHOW THAT MORE THAN 6% HAVE DEFECTS.

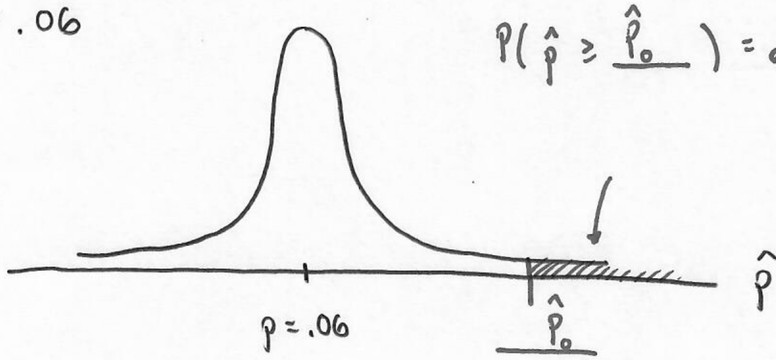
YOU GATHER A SAMPLE OF 1,000 PRODUCTS AND TEST THEM
FOR DEFECTS.

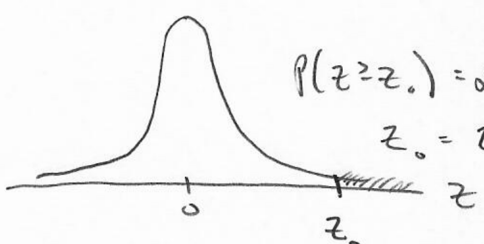
- STATE NULL HYPOTHESIS.
- STATE ALT. HYPOTHESIS.
- USING $\alpha = .01$, FIND THE ACCEPTANCE REGION & REJECTION REGION.

H_0: POPULATION PARAMETER $p = .06$ ← ASSUME

H_A: $p > .06$

$P(\hat{p} \geq \underline{\hat{p}_0}) = \alpha = .01$



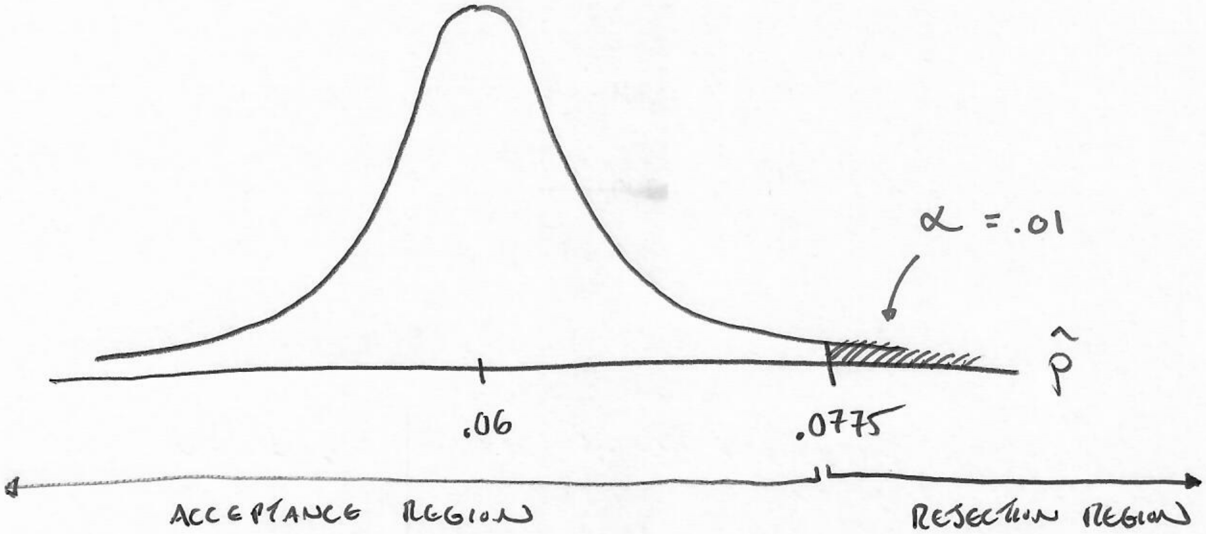


$P(z \geq z_0) = \alpha = .01$

$z_0 = 2.33$

$$z_0 = \frac{\hat{p}_0 - p}{\sqrt{\frac{pq}{n}}} \Rightarrow 2.33 = \frac{\hat{p}_0 - .06}{\sqrt{\frac{(.06)(.94)}{1000}}}$$

$$\hat{p}_0 = .06 + 2.33\sqrt{\frac{(.06)(.94)}{1000}} = .0775$$



IF WE GET $\hat{p} \leq .0775$
WE FAIL TO REJECT H_0,
i.e. WE ACCEPT H_0.

IF WE GET $\hat{p} \geq .0775$
WE REJECT THE NULL HYP H_0
IN FAVOR OF ALT. HYP. H_A.

2 TYPES OF ERRORS WHEN PERFORMING HYPOTHESIS TESTS:

TYPE I: WE REJECT H_0 WHEN IN FACT H_0 IS TRUE.

$P(\text{TYPE I ERROR}) = \alpha$.

SET THIS IN ADVANCE

TYPE II: WE FAIL TO REJECT H_0 WHEN IT IS ACTUALLY FALSE.

MUST BE EQUALITY

H_0: (DEFAULT ASSUMPTION) $\mu = 670$

H_A: (WHAT YOU WANT TO SHOW) $\mu > 670$.

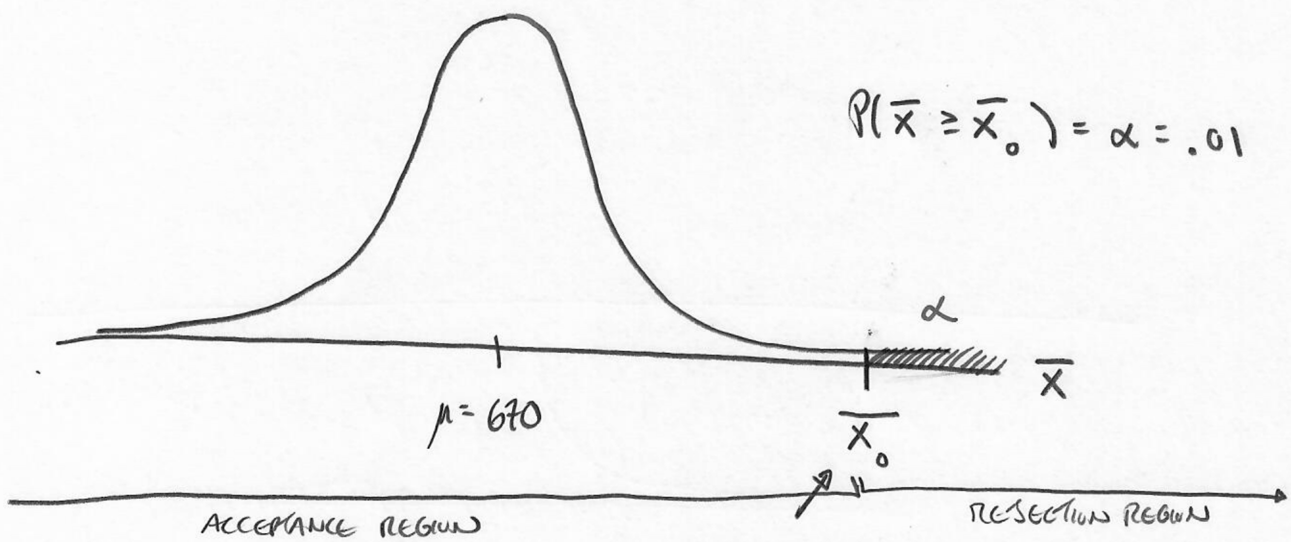
CLT ✓

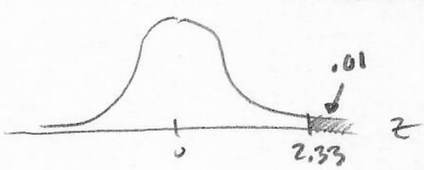
DISTRIBUTION FOR $\bar{X}$ IS APPROX NORMAL (SAMPLE SIZE $n = 40$)

WITH MEAN $\mu = 670$ (H_0)

$$S.E. = \frac{\sigma}{\sqrt{n}} \approx \frac{s}{\sqrt{n}} = \frac{102}{\sqrt{40}}$$

$P(\bar{X} \geq \bar{X}_0) = \alpha = .01$

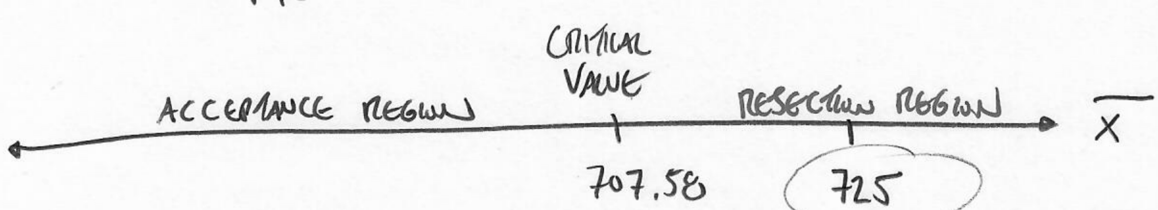




∴ FIND $\bar{X}_0$ AND OBSERVE WHETHER OR NOT $\bar{X} \geq \bar{X}_0$.

$$\bar{X}_0 = \mu + 2.33 \text{ S.E.}$$

$$\bar{X}_0 = 670 + 2.33 \frac{102}{\sqrt{40}} = 707.58$$



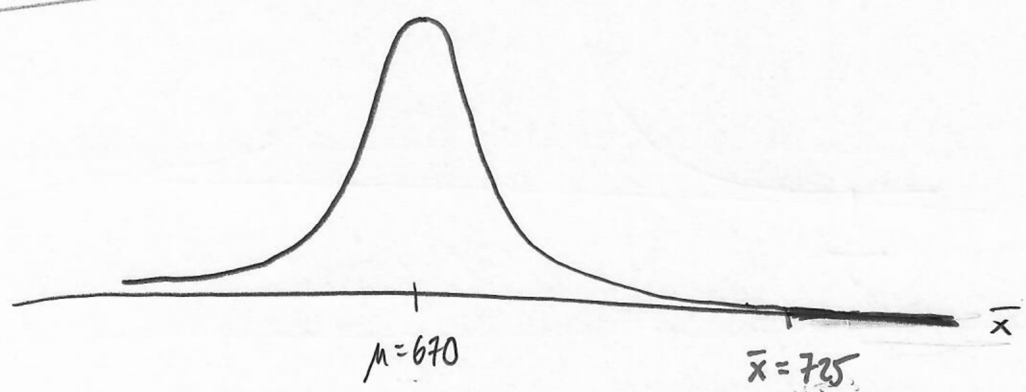
REJECT H_0 IN FAVOR OF H_A: $\mu > 670$

ANOTHER WAY : p-VALUE

Def: THE p-VALUE IS THE PROBABILITY OF OBSERVING A TEST STATISTIC AS EXTREME OR MORE EXTREME THAN THE OBSERVED VALUE, ASSUMING H_0 IS TRUE.

IF $p \le \alpha \Rightarrow$ WE REJECT H_0 IN FAVOR H_A.

IF $p > \alpha \Rightarrow$ WE ACCEPT H_0.



p-VALUE FOR $\bar{x} = 725$: $\underbrace{P(\bar{x} \ge 725)}_{\text{HOW RARE IS THIS?}} = 1 - P\left(z \le \dfrac{725 - 670}{\frac{102}{\sqrt{40}}}\right)$

PROB < α ?

$= \underline{\underline{.0003}}$

p-VALUE $.0003 < \alpha = .01$

NOTE: THE 2 METHODS ARE EQUIVALENT: $\bar{x}$ IN REJ. REGION : $p \le \alpha$

$\bar{x}$ IN ACCEPTANCE REGION : $p > \alpha$

ex.

NULL $H_0 : \mu = 3300$ (ASSUMPTION)

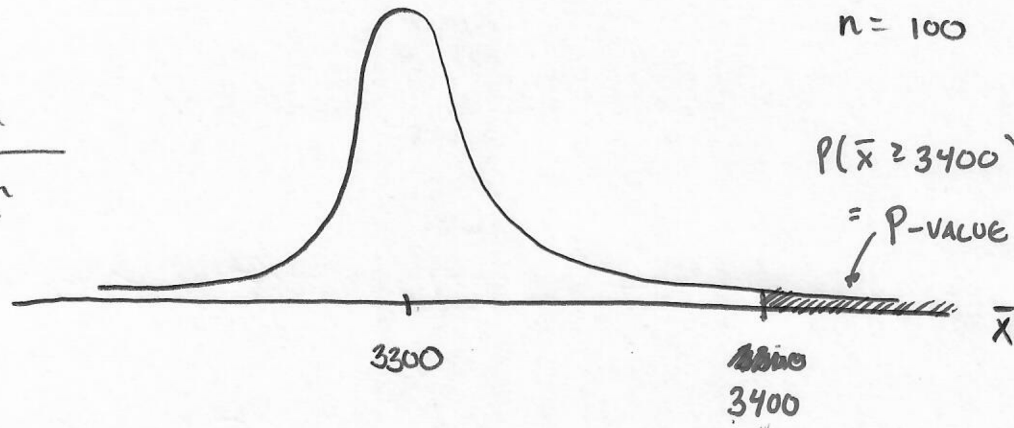
ALT. $H_A : \mu > 3300$

$\bar{x} = 3400$

$s = 1100$

$n = 100$

$$z = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$



$$P(\bar{x} \geq 3400) = P\left(z \geq \frac{3400 - 3300}{\frac{1100}{\sqrt{100}}}\right)$$

$$= 1 - P(z \leq .91) = 1 - .8186 =$$

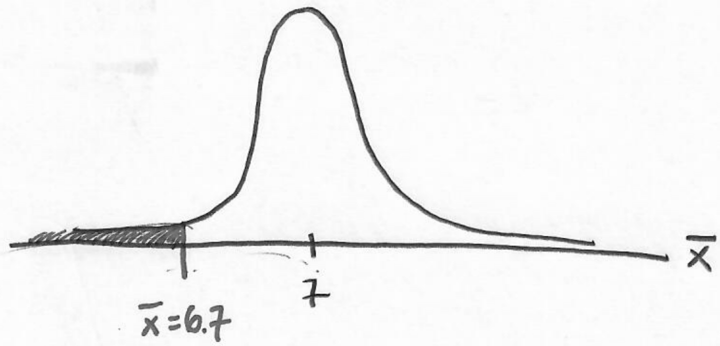
p - VALUE $= .1814$ p-VALUE $> \alpha = .05$

$\Rightarrow$ ACCEPT H_0

DO NOT HAVE STRONG EVIDENCE THAT $\mu > 3300$.

9.14 $H_0: \mu = 7$

$H_A: \mu < 7$



p-VALUE: $P(\bar{x} \leq 6.7) = P\left(z \leq \dfrac{6.7 - 7}{\frac{2.7}{\sqrt{80}}}\right)$

p-VALUE $= P(z \leq -.99) = .1611$.

p-VALUE $> \alpha$

$.1611 > .05$

$\Big\}$ ACCEPT H_0 $\mu = 7$

(FAIL TO REJECT)

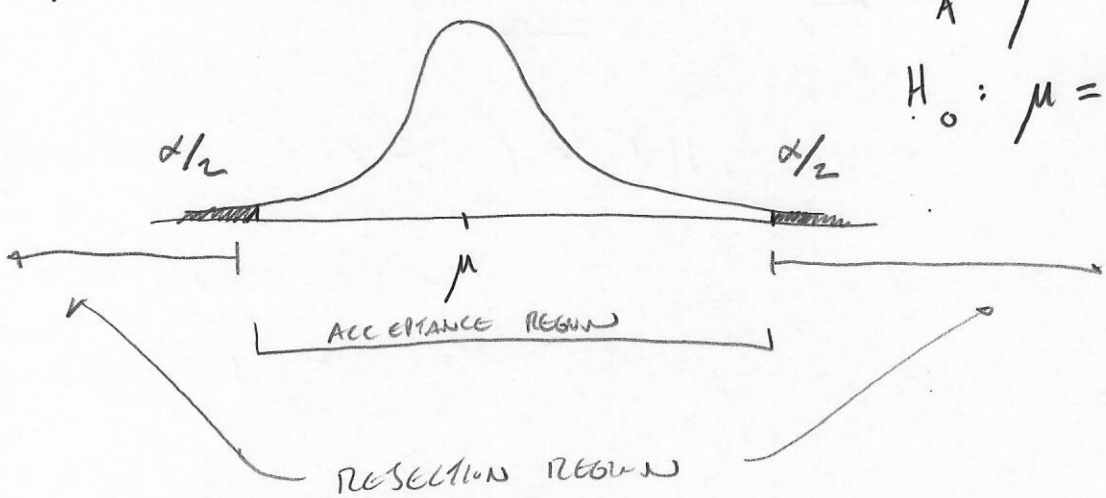
SO FAR, WE'VE FOCUSED ONE ONE-TAILED TESTS OF HYPOTHESES.

$H_A : \mu < \#$

or $H_A : \mu > \#$

NOW, TWO-TAILED TESTS OF HYPOTHESES: $H_A : \mu \neq \#$

$H_0 : \mu = \#$



p-VALUE FOR SAMPLE STATISTIC x_0 WITH z-VALUE z_0

$$= P(z \leq -|z_0|) + P(z \geq |z_0|)$$

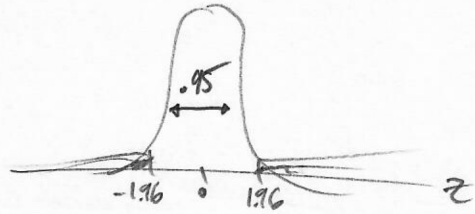
9.13 $H_0: \mu = 80$

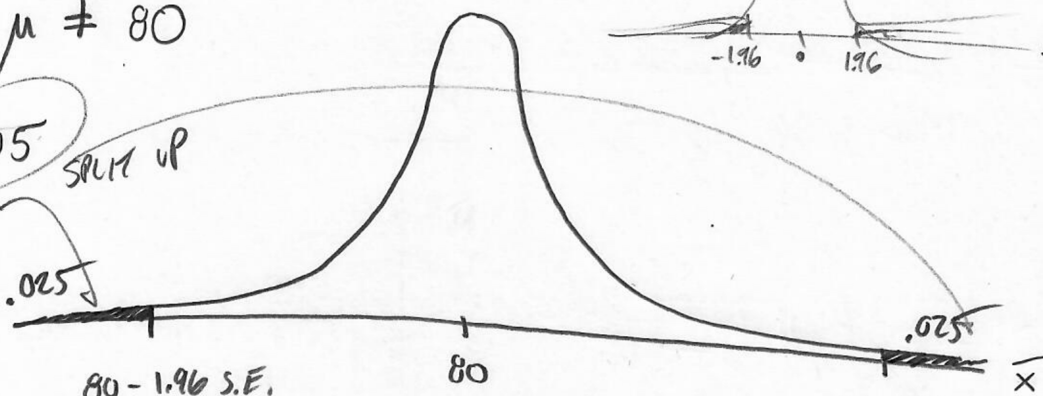
TWO - TAILED

$H_A: \mu \neq 80$

$\alpha = .05$

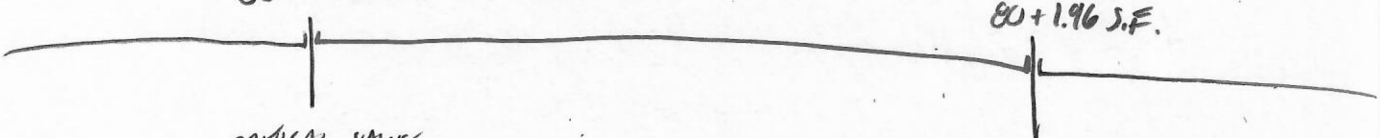
SPLIT UP





80 - 1.96 S.E.

80 + 1.96 S.E.



CRITICAL VALUE

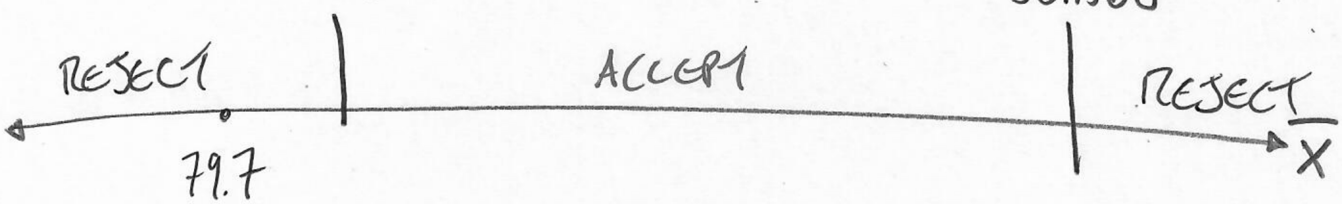
$80 - 1.96 \frac{.8}{\sqrt{100}}$

$= 79.8432$

CRITICAL VALUE

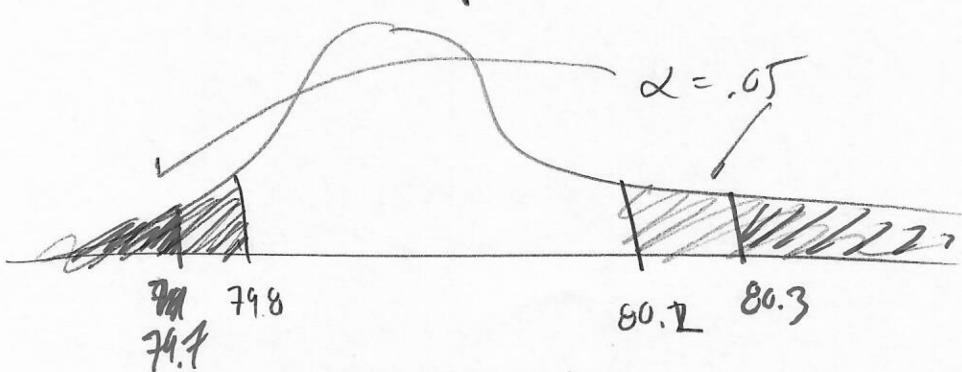
$80 + 1.96 \frac{.8}{\sqrt{100}}$

$= 80.1568$



SINCE $\bar{X} = 79.7$ IS IN THE REJECTION REGION

$\left(P(\bar{X} \leq 79.7) + P(\bar{X} \geq 80.3) \leq \alpha \right)$



FOR FINAL EXAM

§9.1-9.3 CALCULATE p-VALUE FOR 1-TAILED HYPOTHESIS TEST.

9.1 a

9.3 a

9.4 a, c

9.10

9.13

9.14

9.15

9.17